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Abstract

Through extending a standard Grossman and Stiglitz (1980) noisy rational expectations economy by a heterogeneous signal structure with signal-specific differences in uncertainty, we show that price momentum as well as reversal are not intrinsically at odds with rational behavior. Differences in information quality in combination with asymmetric information lead to an under- and over-reaction in equilibrium prices. We derive our results in a standard setup in which information asymmetry is mimicked by access to the realization of a certain signal including its quality, as well as in an environment in which signal quality is the only source of information asymmetry. Both scenarios support price patterns like momentum and reversal in a competitive rational expectations equilibrium without implying investor irrationality. Furthermore, we are able to show that in equilibrium it is always rational for agents to draw inference on their information sets, even in a "second-best" way. By "second best", we refer to the notion that the way in which agents process their information might result in systematic mistakes owing to the existence of asymmetric information regarding signal precision.

JEL Classification: D53; D82; G12; G14

Keywords: General Equilibrium; Asymmetric Information; Asset Pricing; Market Efficiency

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*Any errors are solely my responsibility.

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1 Introduction

This paper targets one of the most pervasive but probably least understood mechanisms in the process of stock price formation, which is rather in tension with the classical economic notion of rationality and efficient markets: the emergence of price momentum and reversal. The literature considers reversal as market over-reaction, while momentum is interpreted as both a consequence of market under-reaction (e.g., Jegadeesh & Titman, 1993) as well as the result of market over-reaction (e.g., Lee & Swaminathan, 2000). We provide an explanation for the existence of momentum (reversal) patterns in a competitive rational expectations model. In our model, momentum and reversal occur in the aftermath of the initial under- and over-reaction of equilibrium prices.

The main building block is a noisy rational expectations economy in the style of Grossman and Stiglitz (1980). In this setting, risk-averse agents trade a risky asset based on private as well as public information. We extend the model by adding an additional layer of information allowing for different types of private information. Hence, we assume that private information has two major components. First, the intrinsic value of the information itself, defined by simply being granted access to information. This covers the classical notion of information in the literature, mostly modeled by observing the realization of a private signal. Second, we regard the ability to evaluate the received information appropriately, as an autonomous component. In most models, this constitutes an implicit assumption. We challenge the existing notion of information as a homogeneous signal that everyone involved is always capable to process correctly.

In reality, information is far too diverse and complex as to be regarded as homogeneous. Many different types of information exist, which might each require specific methods of evaluation. For instance, information could differ along the lines of quality or thematic content, to name just two possibilities. It seems appropriate that these distinctions have to be taken into account when trying to process information properly. Our model allows for the suggested diversity within the information structure and agents have to cope with the issue when exploiting their information. We mimic the diversity of information by introducing two different signal regimes: one providing information connected with a rather low uncertainty level and the other with a relatively higher level of uncertainty. These two signal regimes fit the above interpretation of information having a different quality or thematic content. Intuitively, in such a context the optimal reaction to private as well as public information depends on the respective signal regime. Without making the prevailing signal regime public knowledge, it is easy to guess that it is impossible for the price system to convey all information and be strong-form efficient.[1]

[1] An equilibrium price is said to be strong-form efficient if it is a sufficient statistic for all of the

This posits a problem to the agents, who gather their information from the price system.

The lack of awareness concerning the different types of information is at the heart of our model. In the first part of the paper, we show that signals with low variance are responsible for under-reaction, while signals associated with a higher variance are rather connected to over-reaction. In the second part of the paper, we demonstrate that it is actually sufficient to impose uncertainty regarding the signal regime to establish the described price patterns. The underlying value of the signal can be common knowledge. As an illustration, suppose that a risk-averse agent observes a signal, but has no idea about its quality. He will either a priori make a guess and opt for a specific quality or value the information somewhere in between its potential states of quality. Both strategies result in a systematic deviation from optimal behavior given full information. In the latter situation, the agent will always either overstate the quality of the signal in the event of the high-variance signal translating into an over-reaction, or understate the quality of the low-variance signal resulting in an under-reaction. Opting for the low-variance regime ex ante, the agent overstates signal precision if the high-variance regime is in place. By contrast, deciding for the high-variance regime ex ante, he will understate signal precision in the low-variance regime. The described mechanics induce price momentum and price reversal (under- and over-reaction) without relying on further assumptions like behavioral biases or the existence of momentum traders.

We show that given the described signal structure, there exists an equilibrium that fulfills the conditions of a noisy rational expectations equilibrium (REE) and produces the desired price patterns when

- the economy exhibits asymmetric information regarding both types of information, i.e. signal value and signal regime; and
- the economy exhibits asymmetric information only regarding the signal regime.

Furthermore, we prove that in neither of the above-stated scenarios are prices strong-form efficient and able to convey all information inherited in the economy. Additionally, we show that knowing about the type of information and how to correctly use it posits a valuable asset.

The idea to distinguish thematic content along the lines of signal uncertainty is in line with a prominent strand of empirical literature. It combines the idea of Odean (1998) that people on the one hand are prone to place too much trust in information that is low in precision and on the other hand place insufficient trust in information that is high

information in the economy.

in precision, with empirical evidence on price momentum and reversal owing to certain kinds of events. More precisely, evidence of price momentum has been detected for events like earnings announcements (e.g., Bernard & Thomas, 1990), analyst forecast revisions (e.g., L. K. C. Chan, Jegadeesh, & Lakonishok, 1996; Gleason & Lee, 2003) and share repurchases (e.g., Lakonishok & Vermaelen, 1990), to name just a few, while reversal is attributed to events like initial public offerings (Ritter, 1991), acquiring firms in mergers (Agrawal, Jaffe, & Mandelker, 1992) or new exchange listings (Dharan & Ikenberry, 1995). It seems comprehensible that in general events producing momentum carry a different level of inherent uncertainty regarding their future implications than events giving rise to reversals. Earnings announcements or share repurchases state hard facts with respect to the situation of a company compared to announcements of an acquisition or IPO, which could have several different implications and are complex to evaluate precisely.

Among the recent strand of literature attempting to explain price anomalies like momentum and reversal with rational theories, our paper is most closely related to Holden and Subrahmanyam (2002) and Cespa and Vives (2012). They attribute momentum to an increase in information precision. This is comparable to the technical effect when the uncertainty about the low-variance regime resolves. However, the intuitive ideas of the two models are rather different. While our model concentrates on an additional feature of information, Cespa and Vives (2012) assume information precision to increase with the arrival of further fundamental information.

Andrei and Cujean (2017) also build on a Grossman and Stiglitz (1980) type of economy, although their economic intuition as well as their technical setting strongly differ from our model. They model momentum as the consequence of a special type of information diffusion, namely word-of-mouth communication. For this purpose, they introduce a mechanism of information percolation originally developed by Duffie and Manso (2007) into their noisy rational expectations model.

Our model further has a sharp distinction to Banerjee, Kaniel, and Kremer (2009), who reason momentum with the existence of higher-order beliefs, which imply that agents do not use prices correctly. They impose different beliefs, which in the literature is framed that agents "agree to disagree". Upon first glance, it seems rather familiar with our setting of having different signals that could not be distinguished precisely. However, there is an economic difference if material information differs regarding its characteristics or if agents cannot agree on the characteristics of a homogeneous signal.

Another popular method in the literature is to use growth-option models like Berk, Green, and Naik (1999), Johnson (2002) and Sagi and Seasholes (2007). We significantly distinguish our model from this strand of the literature as we solely concentrate on one market and do not rely on the existence of additional derivative markets to establish our results.

Furthermore, our model does not contradict the well-established behavioral literature related to momentum and reversal, (e.g. Barberis, Shleifer, & Vishny, 1998; Hong & Stein, 1999; Daniel, Hirshleifer, & Subrahmanyam, 1998). However, we do not rely on biases that are standard in this strand of the literature to generate the desired price patterns. Nevertheless, these models seem complementary to ours. We could allow for these biases in our model without annihilating the main results regarding the development of prices but rather amplifying them.

In terms of the empirical literature, our model is also able to incorporate the view of W. S. Chan (2003), who argues that investors under-react to informative information signals while over-reacting to signals that are not informative. The model incorporates this setting as a kind of corner solution when increasing the variance in the high-variance regime to a sufficiently high level. Given this calibration, one of the signals is actually non-informative. However, the established equilibrium results do not change given this calibration.

The organization of the paper is as follows. In the next section, we provide a brief overview of the most important features of noisy rational expectations models, as well as how to solve for equilibrium in such a framework. In section 3, we introduce a one-period static benchmark model and prove the existence of a general noisy REE in the defined setup. Section 4 adds an additional trading period and establishes the price patterns of over- and under-reaction in equilibrium prices given more than one trading round. Section 5 presents an extension of the model in which the uninformed agents observe the signal but not its precision. Further equilibrium prices and price dynamics of the extended model are derived and discussed. Concluding remarks are provided in section 6. All derivations and proofs as well as computational details are relegated to the appendix.

2 General overview of the model

This section first rather generally outlines the most important features of REE models and how they are connected to the model analyzed in this paper. Subsequently, we provide a brief informal overview of the most important features of the model that we present.

2.1 Rational Expectations Equilibria (REE)

The model is set up as a classical noisy REE model and is closely connected to Grossman and Stiglitz (1980). In noisy REE models, all agents behave competitively and act as price takers. According to Brunnermeier (2001, p. 66), each group of agents can be thought of as a "continuum of clones" possessing identical private information. Assume an economy with two kinds of assets: a riskless asset with a pay off of one, as well as a risky asset, which has a stochastic pay off. Furthermore, there are two groups of different agents: informed, I and uninformed, U. The informed agents observe a noisy signal S about the pay off of the risky asset, while the uninformed do not. All distributional assumptions made are common knowledge among all agents in the economy. Each agent maximizes his expected utility by submitting demand schedules contingent on his information set. Simultaneously submitting demand schedules allows traders to take prices as given since it enables agents to submit a specific demand for each possible price. In addition to the informed and uninformed agents, there exists noisy aggregate supply u, which is often referred to as noise traders' demand in the literature. The noisy aggregate supply prevents the price system in the economy from being fully revealing (and is responsible for the "noisy" in the term noisy REE). Denoting the demand of the informed agents by $X_I(S,p)$ and the demand of the uninformed agents as $X_U(p)$, a REE equilibrium exists if both agents submit demand schedules that maximize their expected utilities $E\left[U_I(X_I(S,p))\right]$ and $E\left[U_U(X_U(p))\right]$ and there exists a price, p, which clears the market. In equilibrium, the following two conditions have to hold.

1. Market Clearing:

$$X_I(S,p) + X_U(p) = u. \tag{1}$$

2. Agents Optimization:

$$\begin{aligned} X_I(S,p) &\in \arg\max E\left[U_I(X_I(S,p))|S,p\right], \\ X_U(p) &\in \arg\max E\left[U_U(X_I(p))|p\right]. \end{aligned} \tag{2}$$

A possible closed-form REE solution to the above-outlined setting is usually derived applying a five-step procedure. First, conjecture a price function, which is simply a mapping of the information sets of all agents into the price space. Second, derive the posterior beliefs of all agents regarding the unknown variables of the economy, taking the price conjecture of step one as given. Third, determine each agent's optimal demand, given the price conjecture and the posterior beliefs. Fourth, impose market clearing. Finally, impose rationality by checking whether the conjectured price function of step 1 coincides with the actual price function derived in step 4. If the price conjecture is confirmed to be self-fulfilling in step 5, the solution worked out constitutes an REE. However, for some economies REEs do not exist.[2] For their existence, it is crucial that

[2]For some economies, REEs do not exist. A detailed discussion of the necessary conditions for the existence of equilibria in rational expectations models is given in O'hara (1995).

agents recognize the information structure, e.g. that prices convey information and that the information is measurable in excess demand functions.

The setting examined in this paper distinguishes itself from the above-outlined "classical" framework by introducing heterogeneous information. The signal structure is no longer limited to one kind of noisy signal, but rather allows for different kinds of signals all having different characteristics. More specifically, we allow for two types of noisy signals that differ regarding their quality. We introduce one signal that is rather precise and reliable and another signal that incorporates more uncertainty and hence is less precise. Which type of signal actually occurs is determined exogenously and is not within the responsibility of the agents.

One way to think of this in reality is having different sources of information that discriminate concerning their quality, e.g. due to the skills of the analyst filing the information. Since not every analyst covers every topic, the agent's influence on the quality of the information that he receives is limited. Nevertheless, to process the information optimally, it is essential to have an idea about its quality. Another possible interpretation is the existence of different kinds of events, which are at the basis of the information incorporated in the signal. The more precise signal could be e.g. thought of as an event like an earnings announcement. Earnings announcements depict rather precise information and thus they are assumed to be straightforward to analyze as they offer limited room for interpretation. On the other hand, M&A announcements could be exemplified as information containing a higher level of uncertainty. M&A announcements constitute important information, but are very difficult to interpret. Reality has shown that the actual consequences of mergers are a priori hard to predict.

In the remainder of the paper, we refer to the more reliable signal as the signal with lower uncertainty. The situation in which the signal materializes is denoted as the low uncertainty state or regime. The lower quality signal is referred to as the signal with high uncertainty. The economy is labeled as being in the high uncertainty state or regime if this signal occurs. The informed agent receives one of the two signals with an exogenously-specified probability. However, he knows which of the two signals he observes. The information set of the uninformed agent is analyzed along two different scenarios. In the first scenario, the uninformed agent does not receive any signal at all, meaning that he neither knows the value nor the quality of the signal; rather, he simply knows the signal structure of the model, the distributional assumptions and that the informed agent has observed one of two possible signals. In the second scenario, the uninformed agent is able to observe the value of the signal but does not have any information about its type. We analyze both settings separately, the former in sections 3 and 4, the latter in section 5.

In the analysis, we intend to deflect the focus from the existence of equilibrium in general to the implications of the signal structure on the equilibrium price. Nevertheless, we will prove the existence and say something about uniqueness as well as the potential equilibria's most important properties. The focus is placed on the price process implied by the existing equilibria. We concentrate on the possibility to generate price patterns of over- and under-reaction, which are also discussed in the empirical literature. The idea is to explain these patterns by a simple alteration of the signal structure within a noisy REE.

2.2 Informal description of the model

In the economy analyzed in the following, the informed agents conduct their optimization using the information inherited in the signal that they have received. The uninformed do not know the value of the signal, nor do they know if the signal is of the high or low uncertainty type. The supply of the risky asset—which in the literature is labeled noise trader demand or aggregate endowment—is random. We assume that it is not observable by the uninformed agents. However, this is not due to the usual reason of preventing the price system from being fully revealing. Owing to the heterogeneous signal structure incorporating two different signal regimes, the price system would not be revealing even if the uninformed were able to observe aggregate demand. The technical reason is that the mapping from the signal space into the price space is not "one-to-one" and thus not invertible. This can be thought of as the informed agents having two different demand schedules for each realization of the signal, depending on the signal type. The uninformed do not know with which of the two demand schedules of the informed they are competing. It is intuitively reasonable that a high-quality signal should not support the same price as a low-quality signal, although they might have the same numeric value. This mechanism is independent of noise trader demand. As long as the uninformed agents are unable to somehow infer the state of the economy, the price system is not fully revealing in the classical notion.[3] However, if noise traders' demand were public knowledge, the uninformed could trick the system by playing a clever strategy. They could simply place a market order and ask exactly their relative share of the observed noise trader demand. As the proportion of informed and uninformed agents is common knowledge, such a strategy is feasible. Doing so would implicitly force the informed agents to clear the market by absorbing their relative share of aggregate demand. The resulting equilibrium would support the full information price by applying the informed agents' price schedule and imposing its corresponding price for the respective demand. The information asymmetry in the economy would not be material for the uninformed agents as the information

[3] Fully revealing in the classical sense means that the uninformed can infer the signal value from simply observing price by inverting the price function (Vives, 2008).

problem could be bypassed by applying this simple trick. Completely prohibiting agents from placing market orders and forcing them to submit demand schedules would be the only way in which this mechanism could be avoided. However, every market order could be framed as a demand schedule that makes the restriction highly artificial and difficult to justify. It is vital to know that the fact that aggregate demand must not be known to the uninformed agent is not to prevent the price system from being fully revealing but rather to motivate the uninformed to somehow evaluate and utilize the existing fundamental information in the economy instead of freeriding on the informed agents' demand schedule.

However, given the described setting, it is not trivial to draw inference from the price system or other statistics of the economy. The signal structure— with two uncertainty regimes—not only breaks apart the one-to-one mapping between price and the expected pay off of the risky asset for the uninformed agents; moreover, it also makes it impossible for the uninformed agents to pin down the price function a priori. This limitation translates to the fact that the uninformed agents' posterior beliefs are not specified and the REE has no tractable closed-form solution. Thus, simply relying on price is insufficient for the uninformed agents to determine their optimal strategy. Facing this severe problem, the uninformed are left with two possibilities: first, they can always decide to ignore the potential information inherited in the price system and submit a demand schedule based on their unconditional expectations or even remain completely away from the market; or second, the uninformed could try to utilize their information set. In order to draw inference from the price system, they have to somehow pin down the mapping between the price and expected pay off before they enter the market and submit their demand schedules. This set of options translates into four intuitive strategies that the uninformed could think of.

1. *unconditional strategy* $\{Un\}$ The uninformed agents do not try to infer information from the price system and simply optimize given their unconditional expectations.

2. *conservative strategy* H The uninformed agents behave as if the signal would always be drawn from the distribution with the higher variance, regardless which distribution the actual signal comes from.

3. *progressive strategy* L The uninformed agents act as if the signal would always be drawn from the distribution with the low-variance, regardless which distribution the actual signal comes from.

4. *mixed strategies* Mx The uninformed make use of their knowledge about the probability of the two different uncertainty regimes and play a mixed strategy by randomizing over the possible uncertainty regimes with the respective probabilities.

A strategy is said to be feasible if its expected utility over final wealth exceeds the utility of the uninformed when staying at home and not participating in the market. In this case of non-attendance, the informed agents would absorb all noise trader demand and the uninformed would not enter the market. At the end of the period, they would be left with the utility over their initial wealth. Hence, the expected utility of the uninformed has to exceed their final utility over their initial wealth. The mixed strategy can be ruled out a priori as it is by definition a linear combination of strategies two and three and thus always dominated by the better of the two strategies.

We solve for equilibrium in the standard five-step approach outlined in section 2.1 above based on the uninformed agents' beliefs implied by their strategy. First, we propose a conjecture of the price function. Second, we derive beliefs of the agents given the conjectured price function as well as the private signal. Third, we derive the optimal demand of the risky asset for the different agents, given their information sets. Fourth, we impose market clearing and solve for price. Finally, we impose rationality by matching the coefficients of the proposed price function with the price function derived in step four.

3 A Static benchmark model

In this section, we present a simple static model with only one trading period that highlights the main mechanisms at work in the economy when introducing a heterogeneous signal structure.

3.1 The Environment

Consider an economy with two kinds of assets: a risk-free and a risky asset. The risk-free asset has a price normalized to one and a pay off of R. The risky asset has a pay off of θ and is distributed $\mathcal{N}(\hat{\theta}, \sigma_\theta^2)$. There exist three different groups of agents. A continuum of two ex-ante identical risk-averse agents with common risk aversion α and CARA utility over wealth at the liquidation date $t = 2$, $U(W_2) = -e^{-\alpha W}$. The risk-averse agents differ along their information sets. A fraction λ of the continuum of risk-averse agents—the informed agents—receive a signal S_i with $i \in \{H, L\}$ about the true value of θ. The two possible noisy signals, S_i with $i \in \{H, L\}$, about the true value of θ are $S_H = \theta + \epsilon_H$ and $S_L = \theta + \epsilon_L$, and they differ in their noise term, ϵ. Both noise terms are normally distributed with mean zero, but have different variances $\sigma^2_{\epsilon_H}$ and $\sigma^2_{\epsilon_L}$. Their relationship is restricted by the inequality $\sigma^2_{\epsilon_H} > \sigma^2_{\epsilon_L}$. This implies that S_L is a more valuable signal than S_H. It has higher precision and hence incorporates less uncertainty. The informed agents always observe only one signal, either S_H or S_L with probability p and $1 - p$, respectively. However, they know whether the signal that they observe is

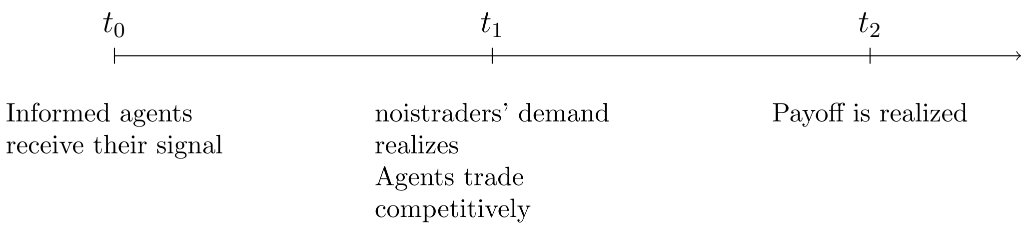

Figure 1: The time line shows the sequence of events in the model. At $t = 0$, the informed agents receive their private signal S_i. In the next step at $t = 1$, financial markets open, noise traders enter and risk-averse agents trade competitively. Uncertainty is resolved at $t = 3$ and the pay off of the risky asset θ materializes

S_H or S_L. The remaining fraction, $(1 - \lambda)$, of the risk-averse agents do not receive any further information about θ and are hence called the uninformed agents. Furthermore, there is a group of noise traders who do not maximize their utility but trade for reasons outside of the model. Their demand typically stems from information that is not of common interest, such as from their need to hedge against endowment shocks or private investment opportunities in an incomplete market setting (Brunnermeier, 2005). Some models also work with the assumption of random aggregate demand, which is technically identical to the noise trader assumption. In order to be consistent throughout this paper, we stick to the notion of noise trader demand.[4]. The noise traders' demand per trading sequence is denoted by u and normally distributed with mean zero and variance σ_u^2. It enters the market in period $t = 1$. The uninformed agents know neither the exact value of S_i nor which kind of signal, S_H or S_L the informed agents have received. Furthermore, they are unaware of the realization of noise traders' demand u. However, they can use all public information to make inference. All probability distributions and other parameters of the model are common knowledge. One trading sequence has three dates $t \in \{0, 1, 2\}$. At $t = 0$, the informed agents receive their signal S_i with $i \in \{H, L\}$ and noise traders' demand, u realizes. At $t = 1$, financial markets open, the noise traders enter the market and the informed and uninformed agents trade competitively. At the final date, $t = 2$ the risky asset pays its liquidating dividend. Figure 1 shows a timeline of the trade protocol.

Signal structure

The terminal value of the risky asset is given by $\theta \sim \mathcal{N}(\hat{\theta}, \sigma_\theta^2)$. The informed traders observe one of two possible noisy signals S_i with $i \in \{H, L\}$ of the terminal value of the risky asset with probability p and $1 - p$, respectively. According to this setup, the signals

[4]The assumption of liquidity or noise traders who trade for reasons outside of the model and do not optimize their utility is common in this literature. For a detailed discussion of possible reasons why liquidity traders trade as well as an examination of the distinction between information of common versus private interest, see Brunnermeier (2001) or O'hara (1995). Furthermore, it is possible to rationalize noise traders as risk-averse hedgers. For a detailed analysis of the topic, see Manzano and Vives (2011) and Medran and Vives (2004)

follow a normal mixture distribution.

$$f(S) = pf_{S_H}(S) + (1-p)f_{S_L}(S) \tag{3}$$

The components of the mixture distribution are defined as follows

$$\begin{aligned} S_L &= \theta + \epsilon_L \qquad \epsilon_L \sim \mathcal{N}(0, \sigma^2_{\epsilon_L}) \quad \text{and hence} \quad S_L \sim \mathcal{N}(\hat{\theta}, \sigma^2_\theta + \sigma^2_{\epsilon_L}) \\ S_H &= \theta + \epsilon_H \qquad \epsilon_H \sim \mathcal{N}(0, \sigma^2_{\epsilon_H}) \quad \text{and hence} \quad S_H \sim \mathcal{N}(\hat{\theta}, \sigma^2_\theta + \sigma^2_{\epsilon_H}), \end{aligned} \tag{4}$$

with $\sigma^2_{\epsilon_H} > \sigma^2_{\epsilon_L}$. θ and S_L as well as θ and S_H are distributed bivariate normal $\mathcal{N} \sim (\boldsymbol{\mu_{S_L}}, \boldsymbol{\Sigma_{S_L}})$ and $\mathcal{N} \sim (\boldsymbol{\mu_{S_H}}, \boldsymbol{\Sigma_{S_H}})$, with

$$\boldsymbol{\mu_{S_L}} = \boldsymbol{\mu_{S_H}} = \begin{pmatrix} \hat{\theta} \\ \hat{\theta} \end{pmatrix} \quad \boldsymbol{\Sigma_{S_L}} = \begin{pmatrix} \sigma^2_\theta & \sigma^2_\theta \\ \sigma^2_\theta & \sigma^2_\theta + \sigma^2_{\epsilon_L} \end{pmatrix} \quad \boldsymbol{\Sigma_{S_H}} = \begin{pmatrix} \sigma^2_\theta & \sigma^2_\theta \\ \sigma^2_\theta & \sigma^2_\theta + \sigma^2_{\epsilon_H} \end{pmatrix}$$

Given the above distributions, one can calculate the values of $E[\theta|S_H],\ Var[\theta|S_H],\ E[\theta|S_L]$ *and* $Var[\theta|S_L]$.[5]

$$E[\theta|S_L] = \hat{\theta} + \underbrace{\frac{\sigma^2_\theta}{\sigma^2_\theta + \sigma^2_{\epsilon_L}}}_{F_L}(S_L - \hat{\theta}) \quad \textit{and} \quad Var[\theta|S_L] = \sigma^2_\theta - \frac{\sigma^4_\theta}{\sigma^2_\theta + \sigma^2_{\epsilon_L}} = \frac{\sigma^2_\theta \sigma^2_{\epsilon_L}}{\sigma^2_\theta + \sigma^2_{\epsilon_L}}$$

$$E[\theta|S_H] = \hat{\theta} + \underbrace{\frac{\sigma^2_\theta}{\sigma^2_\theta + \sigma^2_{\epsilon_H}}}_{F_H}(S_H - \hat{\theta}) \quad \textit{and} \quad Var[\theta|S_H] = \sigma^2_\theta - \frac{\sigma^4_\theta}{\sigma^2_\theta + \sigma^2_{\epsilon_H}} = \frac{\sigma^2_\theta \sigma^2_{\epsilon_H}}{\sigma^2_\theta + \sigma^2_{\epsilon_H}}$$

3.2 Optimization of the agents

This chapter guides to the optimization problem faced by the different agents and provides a first intuition of the signal structure's implications on the optimization mechanics of competitive rational expectations models.

The Agent's budget constraint

Each of risk-averse agent is endowed with an initial wealth, W_0, which he can invest in the two different kinds of assets. Because a CARA investor's demand is independent of initial wealth, the actual amount of W_0 does not influence the economics of the model. P_t is the price of the risky asset in period t. The price of the risk-free asset is normalized to one for all periods. Without loss of generality, the pay off of the risk-free asset R is also assumed to be one throughout the paper. Each unit of the risky asset has a pay off of θ

[5]The results are derived applying the projection theorem for jointly normal variables.

"dollars" at the end of the period. $A \in \{I, U\}$ stands for the informed I or uninformed U agent. $D_{t,A}$ is agent A's position in the risk-free asset at time t and $X_{t,A}$ agent A's demand of the risky asset in period t. The initial budget constraint writes

$$W_{0,A} = D_{1,A} + P_1 X_{1,A}. \tag{5}$$

The wealth of the agent in period one in terms of pay off is given by

$$W_{1,A} = RD_{1,A} + \theta X_{1,A}. \tag{6}$$

Replacing the demand for the risky asset $D_{1,A}$ and expressing $W_{1,A}$ in terms of $W_{0,A}$, the final wealth of the agents is

$$W_{1,A} = RW_{0,A} + (\theta - RP)X_{1,A}. \tag{7}$$

As already mentioned, all agents have the same utility function over final wealth $V(W_{1,A})$ of the CARA class,

$$V(W_{1,A}) = -e^{-\alpha W_{1,A}}, \quad \alpha > 0,$$

where α is the coefficient of absolute risk aversion. Each agent desires maximizing his expected utility conditional on his information set.

Informed Agent

The informed trader maximizes his expected utility conditional on his information set, $\mathcal{F}^I = \{S_i\}$, which comprises the realization of the signal, S_i, with $i \in \{H, L\}$

$$E[V(W_{1I})|S_i] = E[-e^{-\alpha W_{1I}}|S_i]. \tag{8}$$

Knowing the distribution of S_H and S_L, W_{1I} is normally distributed conditional on the respective Signal. Using log normal properties [6], we can rewrite equation 8 as

$$E[V(W_{1I})|S_i] = -exp(-\alpha(E[W_{1I}|S_i] - \frac{\alpha}{2}Var[W_{1I}|S_i])). \tag{9}$$

Given the properties of exponential utility, $E[V(W_{1I})|S_i]$ is maximized by maximizing

$$E[W_{1I}|S_i] - \frac{\alpha}{2}Var[W_{1I}|S_i]. \tag{10}$$

[6] If $ln(x) \sim \mathcal{N}(\mu_x, \sigma_x^2)$, then $E[e^x] = e^{\mu + \frac{1}{2}\sigma^2}$

Plugging in W_{1I} from equation 7 yields

$$E[W_{1I}|S_i] = RW_{0I} + (E[\theta|S_i] - RP)X_{I,i} \tag{11}$$

$$Var[W_{1I}|S_i] = X_{I,i}^2 Var[\theta|S_i]. \tag{12}$$

Equations 11 and 12 are due to the fact that W_{0I} and R are known when the agent conducts his optimization. P is treated as if it is known, as the informed agent submits a complete demand schedule rather than a single order. He actually places his demand for each possible price. The optimization problem of the informed agent writes

$$\max_{X_{I,i}>0} \quad RW_{0I} + (E[\theta|S_i] - RP)X_{I,i} - \frac{\alpha}{2} X_{I,i}^2 Var[\theta|S_i]. \tag{13}$$

The resulting FOC is

$$E[\theta|S_i] - RP - \alpha X_{I,i} Var[\theta|S_i] = 0, \tag{14}$$

and informed demand is given by

$$X_{I,i} = \frac{E[\theta|S_i] - RP}{\alpha * Var[\theta|S_i]}. \tag{15}$$

Uninformed agent

The general mechanism of the optimization of the uninformed agents is independent of their beliefs and hence the strategy they are playing. Essentially, it always follows the same principles. The uninformed agents do not observe the signal, nor do they know the specific distribution that it comes from. However, they know that there is a signal and that it has to be drawn from one of two possible components of a mixture distribution with known parameters. Furthermore, they understand how the demand of the informed is affected by the signal. The uninformed will capitalize these insights to infer some indication about the liquidation value of the asset, given the price that they observe. For this purpose, they have to conjecture a price function based on their information set, $\mathcal{F}^U$. When the uninformed conduct their optimization, besides price, P, they always have access to signed trading volume. The latter denotes the net order flow of the informed and noise traders (combined demand of the informed and noise traders). This information is available, as it is always possible for the uninformed to check the aggregate position in the order book. Signed trading volume is defined as $\upsilon_i = \lambda \frac{E[\theta|S_i] - RP_\theta}{\alpha * Var[\theta|S_i]} - u$. For mathematical convenience and in line with the literature, we introduce another variable called adjusted volume, ν_i, a simple transformation of signed volume.[7] In the setup of this paper, adjusted volume crucially depends on the strategy of the uninformed and whether their strategy matches the true signal state i, or not. The strategy of the uninformed is indicated by j, with $j \in \{Un, H, L\}$. We compute adjusted volume by starting with υ_i, subtracting the mean exogenous supply $\hat{u}$, which in our case is zero, multiplying the result by the constant $\frac{\alpha * Var[\theta|S_j]}{\lambda}$, which depends on the strategy of the uninformed and adding the price $P_{\lambda,ij}$.[8]

With $P_{\lambda,ij}$ denoting the equilibrium price of the risky asset for a given fraction of informed agents λ, a given signal i and a strategy played by the uninformed j, the adjusted volume that the uninformed base their decision on is defined as

$$\nu_i^j = \frac{Var[\theta|S_j]}{Var[\theta|S_i]}(E[\theta|S_i] - P_{\lambda,ij}) - \frac{\alpha * Var[\theta|S_j]}{\lambda}(u - \hat{u}) + P_{\lambda,ij}. \tag{16}$$

If the true state of the signal regime coincides with the strategy of the uninformed, $i = j$, in the following adjusted volume will be defined as ν_i and takes the form as in Grossman and Stiglitz (1980) and Breon-Drish (2015) given by

$$\nu_i^j = \nu_i = E[\theta|S_i] - \frac{\alpha * Var[\theta|S_i]}{\lambda}(u - \hat{u}). \tag{17}$$

According to 17, conditioning uninformed beliefs on price is equivalent to conditioning uninformed beliefs on a linear function of the signal S and aggregate demand u only. If the true state of the signal regime does not match the action of the uninformed, $i \neq j$, this is no longer true and adjusted volume is given by

$$\nu_i^j = \frac{Var[\theta|S_j]}{Var[\theta|S_i]}E[\theta|S_i] - \frac{\alpha * Var[\theta|S_j]}{\lambda}(u - \hat{u}) + \left(\frac{Var[\theta|S_i] - Var[\theta|S_j]}{Var[\theta|S_i]}\right) P_{\lambda,ij}. \tag{18}$$

It is easy to see that ν_i^j can always be framed solely in terms of $\nu_i and P_{\lambda,ij}$, using $\nu_i^j = \nu_i + (\nu_i^j - \nu_i)$

$$\nu_i^j = \nu_i + \frac{Var[\theta|S_j] - Var[\theta|S_i]}{Var[\theta|S_i]}(\nu_i - P_{\lambda,ij}). \tag{19}$$

Knowing the distributional assumptions on S_i and u, one can work out the exact distribution of ν_i as well as the joint distribution of ν_i and θ. A detailed description of the distributions is given in appendix A.1. Furthermore, ν inherits the mixture structure of

[7] The variable ω_λ in the original model of Grossman and Stiglitz (1980) is nothing but adjusted trading volume.

[8] In this setting, signed volume is informationally equivalent to observing price and provides no additional information. Although more general settings exist, signed volume may enhance the information contained in price. See Breon-Drish (2015) for a more detailed elaboration on the issue.

S. Each value of signed trading volume, υ_i can potentially cause two different values of adjusted volume, ν_i^j depending on the strategy of the uninformed and the signal type S_H or S_L. Each of the two signal values gives rise to a different realization of adjusted volume.

The uninformed agent conducts his maximization based on his information set comprising adjusted volume as well as price, $\mathcal{F}^U = \{\nu_i^j, P_{\lambda,ij}\}$.

$$E[V(W_{1U})|\nu_i^j, P_{\lambda,ij}] = E[-e^{-\alpha W_{1U}}|\nu_i^j, P_{\lambda,ij}], \tag{20}$$

Given that θ, W and ν_i are normal and ν_i^j can be always written as a function of ν_i and $P_{\lambda,ij}$, the conditional distribution of W_{1U} given ν_i^j and $P_{\lambda,ij}$ is also normal. Therefore, expected utility can be written

$$E[V(W_{1U})|\nu_i^j, P_{\lambda,ij}] = -exp(-\alpha(E[W_{1U}|\nu_i^j, P_{\lambda,ij}] - \frac{\alpha}{2}Var[W_{1U}|\nu_i^j, P_{\lambda,ij}])). \tag{21}$$

The above equation is maximized by maximizing

$$E[W_{1U}|\nu_i^j, P_{\lambda,ij}] - \frac{\alpha}{2}Var[W_{1U}|\nu_i^j, P_{\lambda,ij}], \tag{22}$$

resulting in the maximization problem of

$$\max_{X_U>0} \quad RW_{0U} + (E[\theta|\nu_i^j, P_{\lambda,ij}] - RP)X_U - \frac{\alpha}{2}X_U^2 Var[\theta|\nu_i^j, P_{\lambda,ij}]. \tag{23}$$

The FOC is given by

$$E[\theta|\nu_i^j, P_{\lambda,ij}] - RP - \alpha X_U Var[\theta|\nu_i^j, P_{\lambda,ij}] = 0 \tag{24}$$

and the demand of the uninformed writes

$$X_U = \frac{E[\theta|\nu_i^j, P_{\lambda,ij}] - RP_{\lambda,ij}}{\alpha * Var[\theta|\nu_i^j, P_{\lambda,ij}]}. \tag{25}$$

In this paragraph, we show that the general mechanism of the optimization of the uninformed agents is independent of their beliefs and hence the strategy that they are playing. Like the informed agent, the uninformed knows W_{0I} and R and acts as if he knew $P_{\lambda,ij}$, when conducting his optimization. Due to the CARA utility structure, uninformed demand is independent of the initial endowment.

3.3 Equilibrium price

The maximization reveals that the demand function of the informed depends on the signal regime, while the demand function of the uninformed agents additionally depends on their strategy. The demand functions write

$$X_I = \frac{E[\theta|S_i] - P_{\lambda,ij}}{\alpha * Var[\theta|S_i]} \qquad and \qquad X_U = \frac{E[\theta|\nu_i^j, P_{\lambda,ij}] - P_{\lambda,ij}}{\alpha * Var[\theta|\nu_i^j, P_{\lambda,ij}]}. \tag{26}$$

Again, i denotes the actual signal regime and j indicates the strategy of the uninformed agents. We imply market clearing with λ being the fraction of informed and $(1-\lambda)$ the fraction of uninformed agents such that the demand of the risky asset equals supply

$$\lambda X_I + (1-\lambda) X_U = u. \tag{27}$$

Substituting 26 into 27 yields

$$\lambda \frac{E[\theta|S_i] - P_{\lambda,ij}}{\alpha * Var[\theta|S_i]} + (1-\lambda) \frac{E[\theta|\nu_i^j, P_{\lambda,ij}] - P_{\lambda,ij}}{\alpha * Var[\theta|\nu_i^j, P_{\lambda,ij}]} = u. \tag{28}$$

Solving 28 for $P_{\lambda,ij}$, if $i = j$ yields

$$P_{\lambda;i,j=i} = P_{\lambda,i} = \frac{\frac{\lambda}{\alpha Var[\theta|S_i]} E[\theta|S_i] - u + \frac{(1-\lambda)}{\alpha Var[\theta|\nu_i]} E[\theta|\nu_i]}{\frac{\lambda}{\alpha Var[\theta|S_i]} + \frac{(1-\lambda)}{\alpha Var[\theta|\nu_i]}} \tag{29}$$

which can be rewritten as a linear function of ν_i

$$P_{\lambda,i} = \frac{\frac{\lambda}{\alpha Var[\theta|S_i]} \nu_i + \frac{(1-\lambda)}{\alpha Var[\theta|\nu_i]} E[\theta|\nu_i]}{\frac{\lambda}{\alpha Var[\theta|S_i]} + \frac{(1-\lambda)}{\alpha Var[\theta|\nu_i]}}. \tag{30}$$

Solving 28 for $P_{\lambda,ij}$, if $i \neq j$ yields

$$P_{\lambda;i,j} = \frac{\frac{\lambda}{\alpha Var[\theta|S_i]} E[\theta|S_i] - u + \frac{(1-\lambda)}{\alpha Var[\theta|\nu_j]} [\hat{\theta} + G_j(1 + \frac{Var[\theta|S_j] - Var[\theta|S_i]}{Var[\theta|S_i]})\nu_i - \hat{\theta})]}{\frac{\lambda}{\alpha Var[\theta|S_i]} + \frac{(1-\lambda)}{\alpha Var[\theta|\nu_j]} (1 + G_j(\frac{Var[\theta|S_j] - Var[\theta|S_i]}{Var[\theta|S_i]})}. \tag{31}$$

Rewritten as a function of ν_i, we get

$$P_{\lambda;i,j} = \frac{\frac{\lambda}{\alpha Var[\theta|S_i]}\nu_i + \frac{(1-\lambda)}{\alpha Var[\theta|\nu_j]}[\hat{\theta} + G_j(1 + \frac{Var[\theta|S_j] - Var[\theta|S_i]}{Var[\theta|S_i]})\nu_i - \hat{\theta})]}{\frac{\lambda}{\alpha Var[\theta|S_i]} + \frac{(1-\lambda)}{\alpha Var[\theta|\nu_j]}(1 + G_j(\frac{Var[\theta|S_j] - Var[\theta|S_i]}{Var[\theta|S_i]})} \tag{32}$$

One can infer from equation 15 that the demand of the informed is not unique in S, but rather can have two different values depending on i. This spills over to signed trading volume υ and materializes in the two different values of adjusted volume described by 17 and 18 for each observation of υ. According to equations 30 and 32, either of the possible νs implies a different price. Hence, the mapping from S to P, $P(S)$ is not single valued and $P(S)^{-1}$ does not exist. Given this structure, it is impossible for the uninformed to infer the signal regime by simply observing price.

Solving the model for the uninformed playing the conservative strategy (H)

In this paragraph, we work through the above-described mechanics in further detail and try to give some intuition by solving the model for the scenario when the uninformed plays the conservative strategy H. Applying this strategy, the uninformed agent acts as if the signal would only be drawn from the mixture component with the high-variance; hence, as if S_H were the only possible signal in the economy. If the true signal corresponds to S_H, the uninformed gets everything right and the resulting equilibrium is analogous to that in Grossman and Stiglitz (1980). The economy is described by the following equations:
Signed volume is given by

$$\upsilon = \lambda \frac{E[\theta|S_H] - RP_{\lambda,H}}{\alpha * Var[\theta|S_H]} - u, \tag{33}$$

adjusted volume by

$$\nu_H = E[\theta|S_H] - \frac{\alpha * Var[\theta|S_H]}{\lambda}(u - \hat{u}). \tag{34}$$

The equilibrium price is

$$P_{\lambda,H} = \frac{\frac{\lambda}{\alpha Var[\theta|S_H]}\nu_H + \frac{(1-\lambda)}{\alpha Var[\theta|\nu_H]}E[\theta|\nu_H]}{\frac{\lambda}{\alpha Var[\theta|S_H]} + \frac{(1-\lambda)}{\alpha Var[\theta|\nu_H]}}. \tag{35}$$

However, if the true signal is S_L and the uninformed acts as if the signal were S_H, he makes a mistake. The mistake materializes by the conversion from signed volume to adjusted volume. Adjusted volume in case of the true signal is S_L, but the uninformed plays S_H, which is labeled as ν_L^H and is defined as

$$\nu_L^H = \frac{Var[\theta|S_H]}{Var[\theta|S_L]}(E[\theta|S_L] - P_{\lambda;L,H}) - \frac{\alpha * Var[\theta|S_H]}{\lambda}(u - \hat{u}). \tag{36}$$

Knowing $\nu_L = E[\theta|S_L] - \frac{\alpha * Var[\theta|S_L]}{\lambda}(u - \hat{u})$ and $Var[\theta|S_H] > Var[\theta|S_L]$, it is obvious that $\nu_L^H > \nu_L$. Thus, ν_L^H can be framed in terms of ν_L, as $\nu_L^H = \nu_L + (\nu_L^H - \nu_L)$. Using equation 19,

$$\nu_L^H = \nu_L + \frac{Var[\theta|S_H] - Var[\theta|S_L]}{Var[\theta|S_L]}(\nu_L - P_{\lambda;L,H}). \tag{37}$$

The resulting equilibrium price according to 32 is

$$P_{\lambda;L,H} = \frac{\frac{\lambda}{\alpha Var[\theta|S_L]}\nu_L + \frac{(1-\lambda)}{\alpha Var[\theta|\nu_H]}[\hat{\theta} + G_H(1 + \frac{Var[\theta|S_H] - Var[\theta|S_L]}{Var[\theta|S_L]})\nu_L - \hat{\theta})]}{\frac{\lambda}{\alpha Var[\theta|S_L]} + \frac{(1-\lambda)}{\alpha Var[\theta|\nu_H]}(1 + G_H(\frac{Var[\theta|S_H] - Var[\theta|S_L]}{Var[\theta|S_L]})}. \tag{38}$$

The mistake of the uninformed has two direct implications on price. First, the signal, ν_L^H, upon which the conditional expectation is based is larger than it would have been in a setting without asymmetric information. Second, the update factor, G_H, is smaller than it should be due to the overestimated variance of the signal. These two affects work opposite to each other but will never offset each other exactly as the effect owing to G_H is stronger. This gives rise to the price effects of the model stated in theorem 4.1.

3.4 Utility of the agents in equilibrium

The main emphasis of this paper is on studying the price dynamics incurred by heterogeneous information. We do not primarily focus on the analysis of potential equilibria in the information market, the difference in utility between the informed and uninformed agents or the price of information implied by this difference in utility. We mainly investigate the utility of the uninformed agents—especially the difference in utility—if the uninformed face a world with two different levels of uncertainty versus a world with only one level of uncertainty. Furthermore, it is investigated how the different strategies influence the utility of the uninformed. On can think of this as the price of information having two components. First, the difference between knowing and not knowing the exact value of the signal, hence reflecting the classical information asymmetry between the informed and uninformed agents referred to in the literature. Second, the difference in utility if the uninformed agents face two different uncertainty regimes and do not know which one they are currently in compared to a situation with only one uncertainty regime. In the latter, the uninformed know the prevailing uncertainty regime by construction. The following analysis concentrates on the second component of uncertainty as it determines which strategy the uninformed will play. Nevertheless, for completion we start by investigating the utility of the informed agent.

Utility of informed agents

To calculate the utility of the informed agent, we start at the basis of the utility of the uninformed and calculate the ex-ante expected utility of being informed given the information set of the uninformed. This approach comes with two major benefits: first, it simplifies the comparison between the utility of the informed and uninformed; and second, it makes the calculations technically more tractable. The detailed calculations are relegated to appendix A.2.

According to equation 9, the utility of the informed agent writes

$$E[V(W_{1I})|S_i] = -exp\Big[-\alpha\Big(E[W_{1I}|S_i] - \frac{\alpha}{2}Var[W_{1I}|S_i]\Big)\Big] \tag{39}$$

with W_{1I} being

$$W_{1I} = RW_{0I} + (\theta - RP)X_{I,i},$$

Using 39 and plugging in W_{1I} and $X_{I,i}$ after simplifying, one obtains

$$E[V(W_{1I})|S_i] = -exp\Big[-\alpha RW_{0I}\Big] * exp\Big[-\frac{1}{2}\frac{(E[\theta|S_i] - RP_{\lambda;i,j})^2}{Var[\theta|S_i]}\Big]. \tag{40}$$

As the first part on the right hand side of equation 40 is independent of the signal, the second term has to be key. To determine the ex-ante utility of the informed, we evaluate the expectation given adjusted volume

$$E\left[E[V(W_{1I})|S_i]|\nu_i\right] = V(RW_{0I}) * E\Big[exp\Big(-\frac{1}{2}\frac{(E[\theta|S_i] - RP_{\lambda;i,j})^2}{Var[\theta|S_i]}\Big)\Big|\nu_i\Big]. \tag{41}$$

After some algebra, the solution to equation 41 is given by

$$E\left[E[V(W_{1I})|S_i]|\nu_i\right] = V(RW_{0I})\sqrt{\frac{Var[\theta|S_i]}{Var[\theta|\nu_i]}} * exp\Bigg[-\frac{\Big(E[\theta|\nu_i] - RP_{\lambda;i,j}\Big)^2}{2Var[\theta|\nu_i]}\Bigg]. \quad (42)$$

Notice that CARA utility is negative. Hence, the smaller the ratio $\sqrt{\frac{Var[\theta|S_i]}{Var[\theta|\nu_i]}}$, the higher the utility of the informed. Equation 42 states that the difference between informed and uninformed utility induced by knowing the value of the signal is governed by the ratio of conditional variances, which can be interpreted as the relative value of the quality of information. The more precise the signal of the informed compared to the information carried in adjusted volume, the greater its informational advantage. As the informed do not know a priori which signal regime will materialize, their expected utility is given by a weighted average of both regimes governed by the mixture weights. $E\Big[E[V(W_{1I})|S]\Big]$, combined with the facts on normal mixtures yields

$$E\big[E[V(W_{1I})|S]\big|\nu_i\big] = p * E\big[E[V(W_{1I})|S_H]\big|\nu_i\big] + (1-p)E\big[E[V(W_{1I})|S_L]\big|\nu_i\big] \quad (43)$$

which results in the expression for overall utility of the informed being

$$\begin{aligned} E\left[E[V(W_{1I})|S]\right] = V(RW_{0I})\Bigg(p \cdot \sqrt{\frac{Var[\theta|S_H]}{Var[\theta|\nu_H]}} * E\Bigg[exp\Bigg[-\frac{\Big(E[\theta|\nu_H] - RP_{\lambda;H,j}\Big)^2}{2Var[\theta|\nu_H]}\Bigg]\Bigg] \\ + (1-p) \cdot \sqrt{\frac{Var[\theta|S_L]}{Var[\theta|\nu_L]}} * E\Bigg[exp\Bigg[-\frac{\Big(E[\theta|\nu_L] - RP_{\lambda;L,j}\Big)^2}{2Var[\theta|\nu_L]}\Bigg]\Bigg]\Bigg). \end{aligned} \quad (44)$$

Utility of the uninformed agents

Calculating the utility of the uninformed agents, we approach as follows. We define the utility of the uninformed without uncertainty regarding the signal regime as "first best". This constitutes the max utility level that the uninformed can reach without knowing the signal. Subsequently, we compare the different strategies with the "first-best" level and each other to identify dominant strategies that are feasible for the uninformed. It is always possible to frame the utility of the uninformed as a function of ν_i and $\hat{\theta}$, enabling a closed-form solution and making the calculations much easier to interpret. The ex-ante utility of the uninformed in the respective uncertainty regime can be calculated along the following steps. The detailed calculations are relegated to appendix A.3.

The expected utility of the uninformed agent is defined by

$$E[V(W_{1U})|\nu_i] = -exp\Big[-\alpha\big(E[W_{1U}|\nu_i] - \frac{\alpha}{2}Var[W_{1U}|\nu_i]\big)\Big] \tag{45}$$

with W_{1U} being

$$W_{1U} = RW_{0U} + (\theta - RP)X_U,$$

Using 45, plugging in W_{1U} and X_U and simplifying defines expected utility as

$$E\left[E[V(W_{1U})|\nu_i]\right] = -exp\left[-\alpha RW_{0U}\right] *$$
$$E\left[exp\left[-\frac{E[\theta|\nu_i^j, P_{\lambda;i,j}] - RP_{\lambda;i,j}}{\alpha * Var[\theta|\nu_i^j, P_{\lambda;i,j}]}\left((E[\theta|\nu_i] - RP_{\lambda;i,j}) - \frac{1}{2}\left(E[\theta|\nu_i^j, P_{\lambda;i,j}] - RP_{\lambda;i,j}\right)\frac{Var\left[\theta|\nu_i\right]}{Var[\theta|\nu_i^j, P_{\lambda;i,j}]}\right)\right]\right] \tag{46}$$

The expression in the first line of 46 is a constant and in the second line the term in the exponential can be rewritten as a linear function of $(\nu_i - \hat{\theta})^2$. Hence, expected utility of the uninformed can be calculated analytically and in closed form, by plugging in $P_{\lambda;i,j}$, $E[\theta|\nu_i^j, P_{\lambda;i,j}]$, $E[\theta|\nu_i]$, $Var[\theta|\nu_i^j, P_{\lambda;i,j}]$ and $Var\left[\theta\middle|\nu_i\right]$ and realizing the fact that the resulting expression follows a central chi-square distribution with one df. For $i = j$, 46 reduces to the standard form given by

$$E[V(W_{1U})|\nu_i] = V(RW_{0U}) * exp\left[-\frac{1}{2}\frac{(E[\theta|\nu_i] - RP_{\lambda;i})^2}{Var[\theta|\nu_i]}\right]. \tag{47}$$

Difference in utility

As the informed agents receive a signal and know the uncertainty regime due to their informational advantage, their expected utility exceeds the expected utility of the uninformed. In order to calculate the difference in utility, we introduce a new variable called $E[V(W_{aux})|\nu_i]$. It describes the expected utility of the uninformed if they were informed about the signal regime, but the price in the economy would still allow for the possibility that the strategy of the uninformed and the true state of the signal do not match. This allows splitting the connection between information and utility into two components: one regarding the value of the observed signal and the second regarding the advantage implied by knowing the signal regime. The new variable is defined as

$$E[V(W_{aux})|\nu_i] = e^{\alpha R}V(RW_{0I}) * exp\left[-\frac{\left(E[\theta|\nu_i] - RP_{\lambda;i,j}\right)^2}{2Var[\theta|\nu_i]}\right]. \tag{48}$$

The difference in expected utility between the informed and uninformed agents is then calculated with the help of the new variable according to

$$E[V(W_{1I})] - E[V(W_{1U})] = E[V(W_{1I})] - E[V(W_{aux})] + (E[V(W_{aux})] - E[V(W_{1U})]) \tag{49}$$

For $i = j$, $E[V(W_{1U})] = E[V(W_{aux})]$ and the utility difference between informed and uninformed is mainly governed by the ratio $\sqrt{\frac{Var[\theta|S_i]}{Var[\theta|\nu_i]}}$. However, if $i \neq j$, the utility difference between the informed and uninformed incorporates the two aforementioned components: first, the ratio $\sqrt{\frac{Var[\theta|S_i]}{Var[\theta|\nu_i]}}$, which governs the informational advantage of the signal S_i compared to the price system ν_i if there were no uncertainty concerning the signal regime; and second, the difference between $E[V(W_{aux})] - E[V(W_{1U})]$, which takes into account the loss in utility of the uninformed for not knowing the uncertainty regime and hence making a strategic mistake when updating conditional on the adjusted volume that they observe. $P_{\lambda;i,j}$ affects both elements of the utility calculation and guarantees a further distinction from the standard results of the literature.[9]

3.5 General equilibrium

Next we determine the optimal strategy of the uninformed facing the described economy and the set of possible actions defined in section 2.2. Furthermore, we investigate whether the actions of the uninformed lead to a REE that meets the conditions stated in equations 1 and 2 and analyze the main properties of the potential equilibrium.

Theorem 3.1. *There always exists a noisy REE that fulfills conditions 1 and 2 contingent on the set of possible strategies described*

1. *in which the uninformed agent plays either strategy L or H*
2. *there exists a threshold level p^* as a function of the parameters of the model $p^* = f(\sigma_{\epsilon_L}, \sigma_{\epsilon_H}, \sigma_\theta, \sigma_u, \alpha, \lambda)$ with the domain $\frac{1}{2} < p^* < 1 \quad \forall \quad \lambda < \frac{1}{2}$, which determines the optimal strategy for the uninformed. It defines an optimal strategy for any parametrization of the economy*
3. *if $p < p^*$ it is optimal for the uninformed to play strategy L*
4. *if $p^* < p$ it is optimal for the uninformed to play strategy H*

Proof. We prove the above theorem in two steps. First, we show that playing strategy H is always superior compared with staying out of the market or participating without

[9]This setup would also allow solving for an equilibrium in the information market. However, as already mentioned, this is beyond the scope of this paper.

trying to learn from the price system. Next, we consider the expected utility of the uninformed for both strategies and demonstrate that the two functions intersect only once. Finally, equating the expected utility of both strategies yields the expression for p^* in terms of the parameters of the model. For the detailed proof, see appendix A.4 $\square$

It seems reasonable that it is beneficial to use information that one possesses but is not 100 percent sure about its quality cautiously rather than not using it at all. Thus, the first part of the theorem is in line with intuition. However, part two appears to be not as obvious. While it seems logical for very high values of p that strategy H dominates strategy L and vice versa, determining the critical values of p ex ante by economic intuition is not straightforward. One possible way of reasoning is along the lines of risk aversion. Confronted with information of rather unknown quality, utilizing this information more cautiously should accommodate the general idea of risk aversion. This argues in favor of playing strategy H. Accordingly, the uninformed get everything right in the high signal regime. In the low signal regime, they base their decisions on the assumption of higher uncertainty and thus discount their information at a higher rate than required. This can simply be interpreted as being overly cautious. Against the backdrop of this line of reasoning, we expect the optimal region supporting strategy H to be larger than that in support of strategy L materializing in the fact of $p^* < \frac{1}{2}$. However, looking at theorem 3.1, this is not true. The parameter space supporting L strictly exceeds the parameter space supporting H. It emerges that the optimal behavior of an agent is not to be cautious, but rather to minimize his relative informational disadvantage compared to the informed agent. As the value of information decreases with its variance, the utility that the informed can extract from his information in the high-variance regime is smaller than in the low-variance regime. Therefore, the informed is able to more efficiently exploit his informational advantage given the low-variance regime. Thus, it is much costlier for the uninformed to make a mistake in the low uncertainty regime compared to the high uncertainty regime, as the informed is able to capitalize on the mistake of the uninformed more aggressively. In everyday language, this translates into the desire that it is beneficial to get the high impact things right while being more laissez-faire with the minor issues. The low-variance regime is more efficient in the way that the informational disadvantage of the uninformed is higher. The high-variance regime is more forgiving when making a mistake.

Numerical comparative statics

In this section, we discuss the consequences of perturbations of the model parameters on the agents' utility and their optimal strategies. The parameters of the model can roughly be divided into two groups: one having primarily a level effect and influencing both strategies in the same direction with rather equal strength, and the other mainly governing

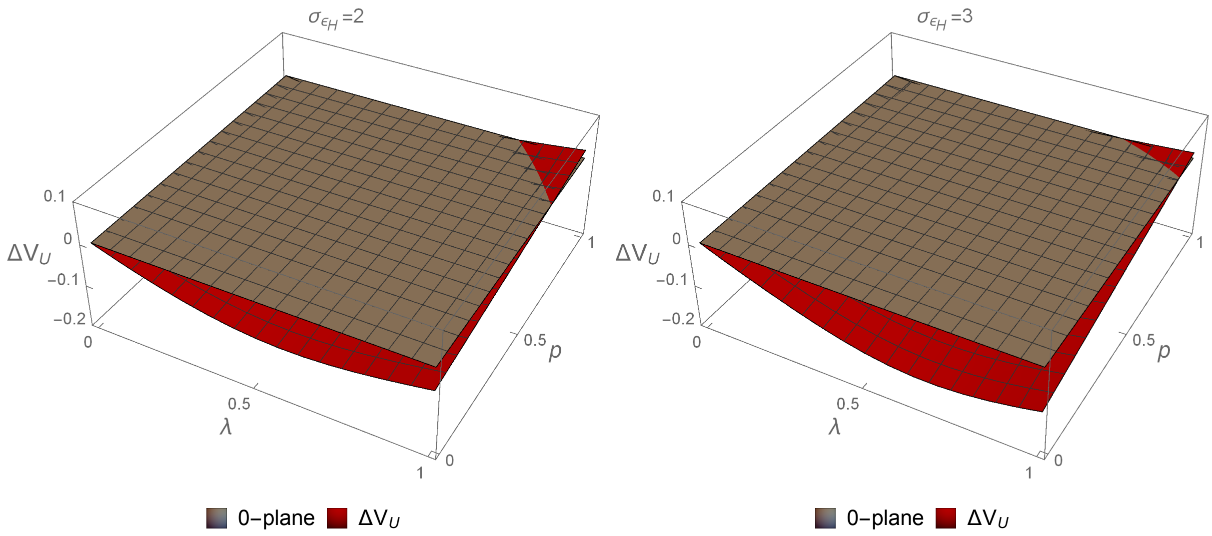

Figure 2: Development of the utility difference of the uninformed between the two possible strategies $V(U_L) - V(U_H)$ with respect to changes λ and p for two different levels of variance in the noise term of the high-variance signal S_H, namely $\sigma_{\epsilon_H} = 2$ and $\sigma_{\epsilon_H} = 3$. The value of the variance in the noise term of the low-variance signal S_L, namely σ_{ϵ_L} and the values of σ_θ, σ_u as well as α are fixed at 1.

the difference between the two strategies and hence defining optimality, p^*. The factors belonging to the latter group—hence primarily determining the utility difference between the two strategies—are the variance of the noise terms of the signal, more precisely their difference, $\sigma^2_{\epsilon_H} - \sigma^2_{\epsilon_L}$, and the variance of the pay off of the risky asset, σ^2_θ. Figure 2 shows the utility difference of the informed agents between playing strategy L and strategy H, $V(W_{U,L}) - V(W_{U,H})$ plotted against the horizontal zero-plane. The graph visualizes the development of the utility difference given an increase in the difference of the signal precision for all possible mixture weights p and different shares of informed agents λ. The variance of the risky pay off, noise trading, risk aversion as well as the noise term of the low-variance signal are held constant at 1, $\sigma^2_u = 1$, $\sigma^2_\theta = 1$, $\sigma^2_{\epsilon_L} = 1$ and $\alpha = 1$. As long as the red plane is below the gray, it is beneficial for the uninformed agent to play strategy L. As soon as the red plane crosses the gray zero-plane from below, it is beneficial to obey strategy H. Notice that due to CARA utility, the more negative the value in the graph, the greater the advantage of playing strategy L and vice versa. One can clearly infer from the two pictures that an increase in the variance of the noise term in the high-variance regime ceteris paribus significantly increases the utility difference between the two strategies and reduces the feasible region of strategy H, hence leading to increasing values of p^*. The mechanism at hand is that an increase in the difference of signal noise by increasing σ_{ϵ_H} reduces the relative informational advantage of the informed in the high-variance regime, while the situation in the low-variance regime remains unchanged. This effects the utility difference between the two strategies in two ways: on the one hand, when playing strategy H, the mistake in the low signal regime is more severe; and on the other hand, the relative benefits of getting the signal right in the high signal regime

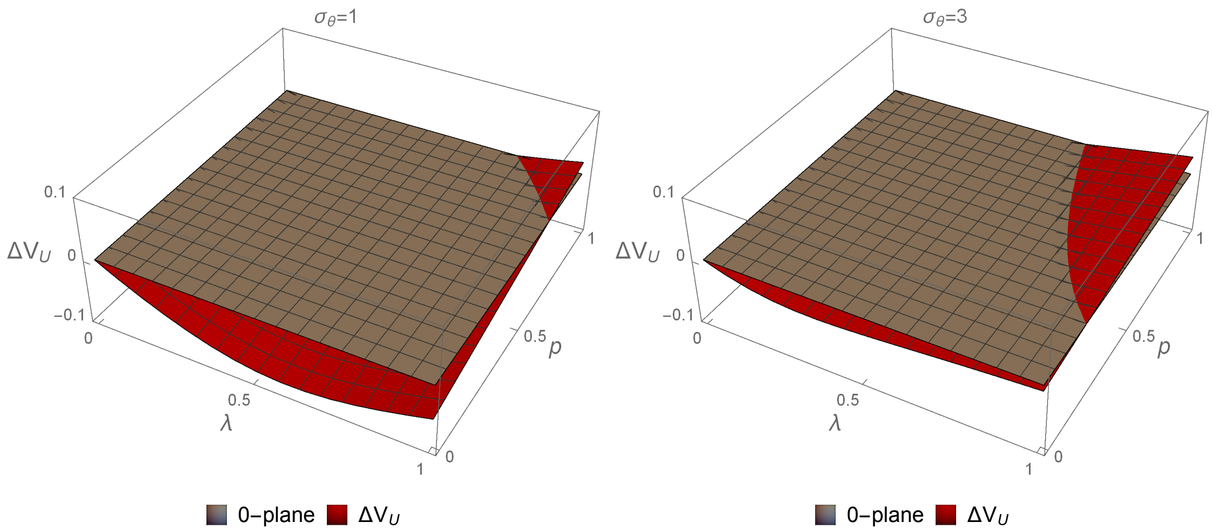

Figure 3: Development of the utility difference of the uninformed between the two possible strategies $V(U_L) - V(U_H)$ with respect to changes λ and p for two different variance levels of the pay off of the risky asset, $\sigma_\theta = 1$ and $\sigma_\theta = 3$. The values of the noise term in the high-variance regime is set to two, $\sigma_{\epsilon_H} = 2$ and the values of σ_{ϵ_L}, σ_u as well as α are fixed at 1.

have decreased as the overall quality of the signal has decreased. The opposite is true for strategy L. Here, the benefits of getting the L signal right remain stable, while the problem of making a mistake in the high-variance regime declines as the scenario as a whole is less favorable for both informed as well as uninformed agents. These dynamics yield the observed increase in the utility difference between the two strategies and the decline in the feasible region of strategy H. Notice that the red area above the zero-plane is significantly smaller in the right picture compared to the left.

The second main force driving the utility difference is the variance of the asset pay off σ_θ^2. An increase in σ_θ^2 ceteris paribus diminishes the difference in utility between the two strategies. This development is depicted in figure 3 for $\sigma_u^2 = 1$, $\sigma_{\epsilon_L}^2 = 1$, $\sigma_{\epsilon_H}^2 = 4$ and $\alpha = 1$. With increasing σ_θ^2, signal noise implicitly decreases relative to the asset variance. This constitutes nothing but a relative increase in signal quality in both signal regimes with the relative difference between the regimes simultaneously decreasing. Therefore, with increasing volatility in the asset pay off, both strategies kind of converge, the utility difference declines and the feasible region of strategy H increases. The development is nicely depicted by the increase of the red area above the zero-plane in the right graph of figure 3 compared with the left.

The remaining variables can be attributed to group one. Their effect is depicted in figure 4 using the overall utility of strategy L as an example. The left graph in figure 4 portrays the influence of an increase in noise trading on the overall utility of the uninformed agents. The other parameters of the model are calibrated as $\sigma_\theta^2 = 1$, $\sigma_{\epsilon_L}^2 = 1$, $\sigma_{\epsilon_H}^2 = 4$ and $\alpha = 1$. An increase in noise trading measured by an increase in σ_u^2 theoreti-

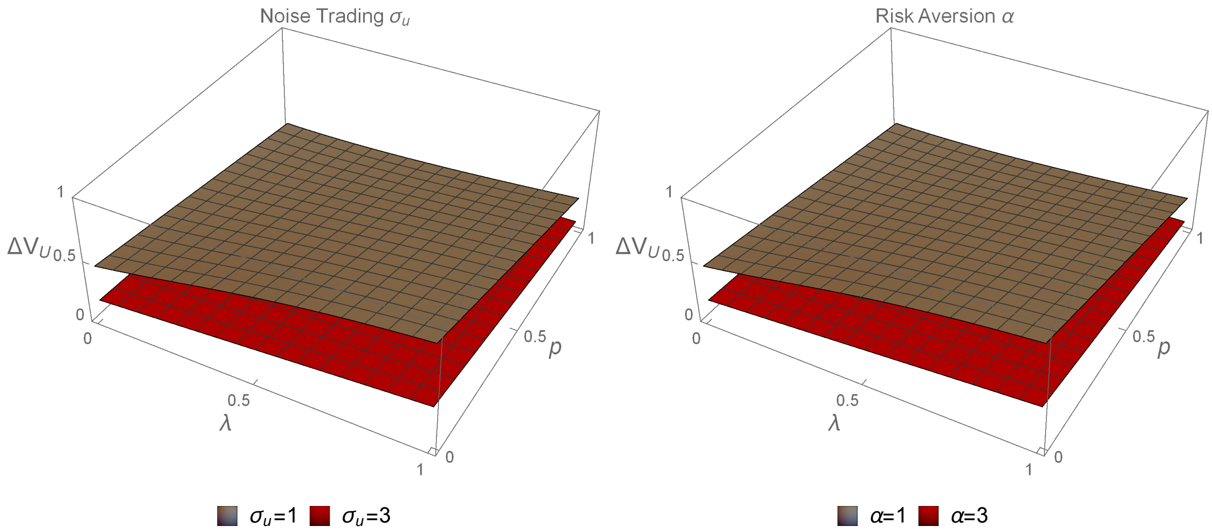

Figure 4: The left graph depicts the development of the utility level of the uninformed agents playing strategy "L" given an increase in the variance of noise trading from $\sigma_u = 1$ to $\sigma_u = 3$. The right graph depicts the development of the utility level of the uninformed agents playing strategy "L" given an increase in risk aversion from $\alpha = 1$ to $\alpha = 3$. The values of the noise term in the high-variance regime is set to two, $\sigma_{\epsilon_H} = 2$. All other parameters of the model σ_{ϵ_L}, σ_θ are fixed at 1.

cally induces two effects: one on the utility difference between informed and uninformed and the other on the overall utility of the uninformed. Concerning the former, it reduces the general informativeness of the price system and hence increases the informational advantage of the informed. However, overall it reduces the equilibrium price and thus generally increases the utility of all agents in the economy. For the uninformed, the latter effect more than overcompensates the decrease in the informativeness of the price system. This can be thought of in two ways. The higher supply reduces the price and as noise traders are not maximizing utility they thus kind of pay the bill in this economy. In turn, the fraction of people who could be exploited by the maximizing agents increases. This positive effect on the utility level shifts the utility plane downwards and is depicted by the difference between the gray and the red plane in the left graph of figure 4. Remember that due to CARA utility, the closer the planes are to zero, the higher the utility of the uninformed agents.

Increasing risk aversion has a similar effect on overall utility. The right graph of figure 4 shows the development of utility for an increase in risk aversion from $\alpha = 1$ to $\alpha = 2$. The rationale behind the presented dynamics is that the higher the risk aversion, the less aggressively the informed agents exploit their informational advantage. Furthermore, the certainty discount on the equilibrium price is higher, which in turn is positive for the uninformed agents as a lower price ceteris paribus increases their overall utility. In both cases, the level effects reduce the overall utility difference between the two strategies.

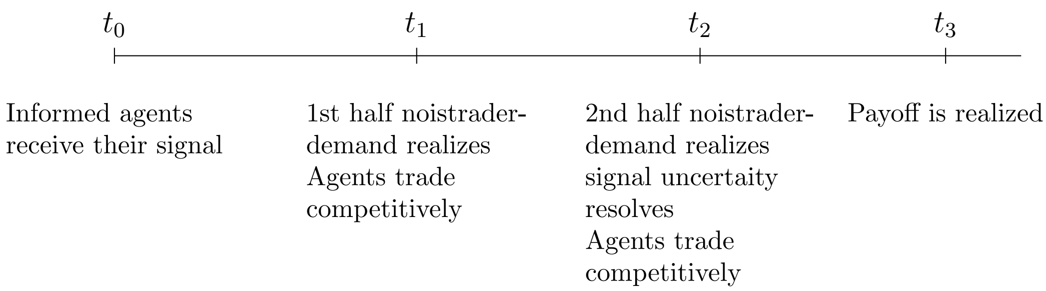

Figure 5: The timeline shows the sequence of events in the model. At $t = 0$, the informed agents receive their private signal S_i. At $t = 1$, the first half of noise traders enters the market and the agents trade competitively. In the next step at $t = 2$, the uncertainty about the signal structure $i = H/L$ is revealed, the second half of noise traders enter the market and agents trade again. Uncertainty is resolved at $t = 3$ and the pay off of the risky asset θ materializes

4 The two-period market

In this section, we extend the static model of section 3 for an additional trading period and examine the equilibrium implications as well as the resulting price dynamics.

4.1 Equilibrium in the two-period market

Paragraph 3 depicts and explains the basic mechanisms of the model. However, in order to investigate price dynamics, one has to look at a model with more than one round of trading. For tractability, we restrict our attention to the case with two trading periods. This allows us to describe and analyze the underlying price dynamics given heterogeneous information. The basic setting is not altered much. In the extended model, the uncertainty regarding the signal regime is resolved after the first round of trading and financial markets open for a second time. Furthermore, noise traders' demand—still denoted by u and normally distributed with mean zero and variance σ_u^2—enters the market by equal parts in two steps. One trading sequence has four dates $t \in \{0, 1, 2, 3\}$. At $t = 0$, the informed agents receive their signal S_i with $i \in \{H, L\}$ and noise traders' demand, u realizes. At $t = 1$, financial markets open, half of the noise traders enter the market and the informed and uninformed agents trade competitively. At the second trading round, $t = 2$ the true state of the signal is revealed to the uninformed agents, and markets open again. The remaining half of the noise traders now enter the market and informed and uninformed agents trade competitively given the new information set of the uninformed. The informed agents do not receive any new information in the second round of trading. At the final date, $t = 3$ the risky asset pays its liquidating dividend. Figure 5 shows a timeline of the trade protocol.

No additional fundamental information regarding the pay off of the risky asset enters the market as the informed agents do not receive an additional signal. Agents maximize their expected end-of-period wealth W_3 and due to CARA utility their wealth level—

expected trading gain from period one—does not influence their optimization in period two. Therefore, the optimization and hence the demand of informed agents does not change in the two-period model as they face an identical information set in both periods. Regarding the uninformed agents, the situation is considerably different: when the signal regime is unveiled in period two, the information set of the uninformed is affected and they incorporate the new information when conducting their optimization. Notice that as they are optimizing end-of-period wealth W_3, their optimal strategy in period one has no implications on their optimal strategy in period two. Accordingly, the first-period behavior of the uninformed does not influence their second-period choices. This is a very nice property as it no longer requires solving the model backwards over two periods but allows us to evaluate each period separately as if it were independent.

In the second round of trading, the signal regime is known to all agents. The equilibrium solutions of the second trading period are given by the equations satisfying $i = j$ and fit a classical noisy REE for both uncertainty regimes.

4.2 Price dynamics

We know the properties of the equilibria in the two periods and the resulting market-clearing equilibrium prices. The next result shows that the behavior of the equilibrium prices from one period to another is systematic and crucially depends on the parametrization of the economy and hence the optimal strategy of the uninformed agents. Overall, two distinct price patterns can be measured across the economy: on the one hand, the economy exhibits momentum, which is triggered by the price movement when observing a low-variance signal but the uninformed opts to see himself in a high-variance regime; and on the other hand, the economy exhibits reversal triggered by the reaction of the uninformed to high-variance signals when he opts to behave as if he were in a low-variance world. There are admissible regions for over- as well as under-reaction patterns in this economy depending on the parameters of the model. The bounds of these regions are mainly governed by the difference in the information quality of the two different signals and the variance of the risky asset. The exact results are stated in the next theorem.

Theorem 4.1. *For each parametrization of the model, there exists a value p^* that determines whether the development of stock prices between the first and second round of trading shows a momentum or reversal pattern.*

1. *If $p < p^*$ the uninformed agent plays strategy L and the economy is in the reversal region;*

    - *Whenever the signal regime is S_H, $|P_{1,S_H}| > |P_{2,S_H}|$ and $cor(\Delta P_{1,S_H} \Delta P_{2,S_H}) < 0$*

- *The overall correlation of price changes between the periods is negative,* $cor(\Delta P_1 \Delta P_2) < 0$

2. *If* $p > p^*$ *the uninformed agent plays strategy* H *and the economy is in the momentum region*

   - *Whenever the signal regime is* S_L, $|P_{1,S_L}| < |P_{2,S_L}|$ *and* $cor(\Delta P_{1,S_L} \Delta P_{2,S_L}) > 0$
   - *the overall correlation of price changes between the periods is positive,* $cor(\Delta P_1 \Delta P_2) > 0$

Proof. See Appendix A.5 □

The economic intuition underlying the price patterns is straightforward. Playing strategy L, the price effect materializes in the presence of the high-variance regime. In such a situation, the actual demand of the uninformed agents exceeds their optimal demand in the prevailing signal regime. The over-reaction owing to a presumably high signal quality shifts up the entire demand schedule of the uninformed, implying a higher demand for each given price. Following the classical mechanics of markets, this excess demand from one side directly translates into a higher market-clearing price in equilibrium compared to a situation without uncertainty regarding the signal regime. As there is no longer signal uncertainty in the second trading period, the over-reaction resolves, leading to a price reversal to its efficient level. Given strategy H, the price effect follows a signal realization from the low-variance regime and the mechanics are vice versa. The uniformed underestimates the quality of his information and reacts too tentatively. The resulting demand induced by the signal is too low, which results in a downward shift of the entire demand schedule. It is easy to see that less demand from the uninformed yields lower prices in equilibrium. The second-period demand will not suffer this bias and thus exceeds the demand in the first period. This results in a higher market price, the initial under-reaction is corrected and a momentum effect occurs.

These price patterns occur without inducing any behavioral biases on the uninformed. As theorem 3.1 shows, the uninformed agents behave fully rationally given their information set and possible choice of actions. They are completely aware of the described mechanism, although ex-ante they are unable to anticipate its occurrence and hence unable to avoid it.

Theorem 4.1 further states that each calibration supports either the over- or under-reaction of prices. However, it is easy to extend the setting in a way that it generally allows for both price patterns independent of the calibration of parameters. As soon as the mixture probabilities are no longer fixed but are allowed to change over time, it is possible to introduce both price patterns in the economy, as the realizations of the

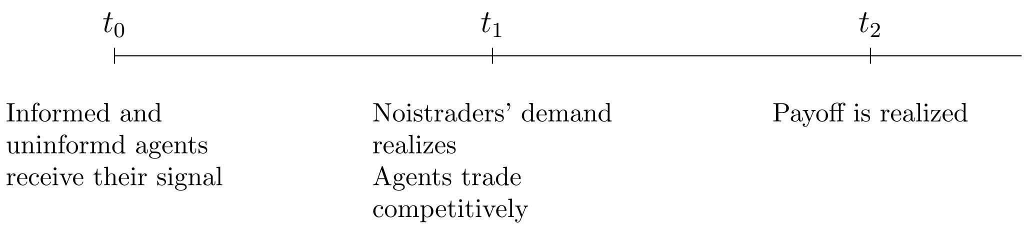

Figure 6: The time line shows the sequence of events in the model when the uninformed receive information about the value of the signal but not about its quality. At $t = 0$, the informed and uninformed agents receive their private signal S_i and S, respectively. In the next step at $t = 1$, financial markets open, noise traders enter and risk-averse agents trade competitively based on their signals. Uncertainty is resolved at $t = 3$ and the pay off of the risky asset θ materializes

mixture weights are common knowledge. Given situations in which the realization equals $p < p^*$ reversal occurs and if $p > p^*$ prices exhibit a momentum pattern. Thus, even the scope of the economy seems limited at first, it is easy to allow for both kinds of price reaction without altering the intuition of the model.

Remark The assumption of the behavior of the noise traders as well as the fact that the informed do not observe a second signal seems a little artificial. However, these assumptions are not essential for the inherent dynamics of the model. The price effect would still be existent but only on average and much more tedious to prove. Therefore, I stick to the above outline, which also makes the basic dynamics of the model much clearer.

5 The Extended model

In the following extension, we study the implications on the equilibrium if the uninformed agents observe the realization of the signal but still remain uninformed regarding the prevailing signal regime.

5.1 General setting

The setting is analogous to that in 3 with the only difference being that the uninformed agents also get to know the realization of the signal S_i. However, they do not know whether the signal that they observe is S_L or S_H. Hence, they simply observe S without having further information. The time line of events is very similar to that described in chapter 3.1 and is given in figure 6

Signal structure

The signal structure is equal to that described at the beginning of chapter 3, with the only difference being that knowing the realization of the signal S_i, the uniformed are now able to make Bayesian inference regarding the probability of the prevailing uncertainty regime. They can assign probabilities to the different mixture components—the part of the distribution the signal comes from—and do not have to stick to the prior weights of the mixture distribution. The prior distribution of S is given by equation 3. The uninformed agents' posterior distribution is given by

$$f(S) = \omega_L f_{S_L}(S) + \omega_H f_{S_H}(S), \tag{50}$$

with the weights of the mixture after observing the realization of S being given according to Bayes' theorem and writing

$$\omega_H = \frac{p f_{S_H}(S)}{(1-p) f_{S_L}(S) + p f_{S_H}(S)} \quad \textit{and} \quad \omega_L = \frac{(1-p) f_{S_L}(S)}{(1-p) f_{S_L}(S) + p f_{S_H}(S)}. \tag{51}$$

ω_L *and* ω_H are the relative probabilities the observed Signal S can be attributed to S_L or S_H.

5.2 Optimization and equilibrium demand

The optimization of the informed does not change as their information set is not affected by the alterations in the setup of the economy. By contrast, the optimization of the uninformed differs due to their additional signal. The signal S is a superior statistic regarding the future pay off of the risky asset compared to the adjusted volume or price. Hence, the uninformed no longer try to draw any inference from price or adjusted volume, but condition directly on the realization of the signal. Although the uninformed know the value of S, the price system of the economy is not fully revealing as they still cannot observe noise trader demand u. Informed demand is still camouflaged by the noise traders. Each value of S can give rise to two different expressions of informed demand X_I, depending on the signal regime. One can think of this as a second layer of uncertainty, impeding that the economy remains fully informationally efficient.

Optimization of the uninformed

The uninformed are optimizing their expected utility. Nevertheless, their ex-ante information set is no longer empty, but rather contains the value of their signal, $\mathcal{F}^U\{S\}$. Thus, they condition their maximization on the signal S that they have observed.

$$E[V(W_{1U})|S] = E[-e^{-\alpha W_{1U}}|S] \tag{52}$$

As the uninformed agents do not know the detailed properties of the signal, they have to update their beliefs about the signal type according to Bayes' theorem, using the weights defined in 51.

$$E[V(W_{1U})|S] = \omega_L E[-e^{-\alpha W_{1U}}|S_L] + \omega_H E[-e^{-\alpha W_{1U}}|S_H] \tag{53}$$

Given that a mixture distribution is a weighted sum of normals, the result for log normal distributions can be used and the respective parts of the expected utility of the uninformed agents can be written as

$$E[V(W_{1U})|S_L] = -exp\left(-\alpha\left(E[W_{1U}|S_L] - \frac{\alpha}{2}Var[W_{1U}|S_L]\right)\right), \tag{54}$$

and

$$E[V(W_{1U})|S_H] = -exp\left(-\alpha\left(E[W_{1U}|S_H] - \frac{\alpha}{2}Var[W_{1U}|S_H]\right)\right). \tag{55}$$

Adding everything up, the expected utility of the uninformed given S writes

$$\begin{aligned} E[V(W_{1U})|S] = &- \omega_L \cdot exp\left(-\alpha\left(E[W_{1U}|S_L] - \frac{\alpha}{2}Var[W_{1U}|S_L]\right)\right) \\ &- \omega_H \cdot exp\left(-\alpha\left(E[W_{1U}|S_H] - \frac{\alpha}{2}Var[W_{1U}|S_H]\right)\right). \end{aligned} \tag{56}$$

Substituting the expressions for $E[W_{1U}|S_L]$, $E[W_{1U}|S_H]$, $Var[W_{1U}|S_L]$ and $Var[W_{1U}|S_H]$, one gets

$$\begin{aligned} E[V(W_{1U})|S] = &- \omega_L \cdot exp\left(-\alpha\left(RW_{0U} + (E[\theta|S_L] - RP)X_U - \frac{\alpha X_U^2}{2}Var[\theta|S_L]\right)\right) \\ &- \omega_H \cdot exp\left(-\alpha\left(RW_{0U} + (E[\theta|S_H] - RP)X_U - \frac{\alpha X_U^2}{2}Var[\theta|S_H]\right)\right). \end{aligned} \tag{57}$$

The maximization problem of the uninformed agents and the corresponding FOC write

$$\begin{aligned} \max_{X_U} \quad &- \omega_L \cdot exp\left(-\alpha\left(RW_{0U} + (E[\theta|S_L] - RP)X_U - \frac{\alpha X_U^2}{2}Var[\theta|S_L]\right)\right) \\ &- \omega_H \cdot exp\left(-\alpha\left(RW_{0U} + (E[\theta|S_H] - RP)X_U - \frac{\alpha X_U^2}{2}Var[\theta|S_H]\right)\right). \end{aligned} \tag{58}$$

FOC:

$$
\begin{aligned}
0 = &\,\omega_L \cdot \left(\alpha \left((E[\theta|S_L] - RP) - \alpha X_U Var[\theta|S_L]\right)\right) exp\left(\Theta_{S_L}\right) \\
&+ \omega_H \cdot \left(\alpha \left((E[\theta|S_H] - RP) - \alpha X_U Var[\theta|S_H]\right)\right) exp\left(\Theta_{S_H}\right)
\end{aligned}
\tag{59}
$$

with

$$
\begin{aligned}
\Theta_{S_L} &= -\alpha\left(RW_{0U} + (E[\theta|S_L] - RP)X_U - \frac{\alpha X_U^2}{2} Var[\theta|S_L]\right) \\
\Theta_{S_H} &= -\alpha\left(RW_{0U} + (E[\theta|S_H] - RP)X_U - \frac{\alpha X_U^2}{2} Var[\theta|S_H]\right)
\end{aligned}
\tag{60}
$$

The uninformed cannot distinguish the different signals' regimes. Therefore, they are unable to maximize their utility dependent on the respective signal regime, but rather have to maximize over their expectation of the signal regime. They are searching for an optimal demand given their signal, which fulfills optimality regardless of the signal regime in place. Contrary to the informed agent, this translates into only one maximization problem and one FOC. Unfortunately, the resulting expression for the FOC has no analytical solution and can only be solved numerically.

Equilibrium demand

Despite the challenges posed above, it is possible to prove existence and characterize the different features of the equilibrium in closed form. As this paper places its focus on the price dynamics that exist in the market, a proof and characterization of equilibrium should be sufficient. The following proposition postulates the existence of optimal demand and pins down its most important characteristics.

Proposition 1.

1. *There exists a unique X_U that fulfills the conditions of optimal demand stated in 2.*
2. *For each signal, the optimal demand of the uninformed agent, X_U, is situated within a specified interval $(\underline{X}, \bar{X})$, with the demand of the informed agents X_{IH} and X_{IL} being the boundaries of that interval.*
3. *For equilibrium demand, the following relations hold. $X_{I,H} > X_U < X_{I,L}, \forall S > 0$ and $X_{I,L} > X_U < X_{I,H}, \forall S < 0$.*

Proof. The proof of proposition 1 is organized in three parts. We first prove that the FOC given by 59 is a continuous function, monotonically decreasing with X_U. Second, we show that optimal demand of the uninformed is bounded by the interval $X_U \in (\underline{X}, \bar{X})$ with the boundaries being $X_{I,L}$ and $X_{I,H}$ and $FOC(\underline{X}) < 0$ and $FOC(\bar{X}) > 0$. Finally, the result follows from the intermediate value theorem. For the detailed proof, see A.6 □

As $X_{I,i}$ is the optimal demand of the informed agents, it always equates the respective signal congruent part of the FOC to zero by definition. After substituting $X_{I,H} = X_U$, the second part of the FOC is zero and one is left with the first part, which constitutes the situation for the opposite signal. Being left with the first part of the FOC, we know from the optimization of the informed that the demand $X_{I,H}$ is too low, as the reaction on the signal information implied by $X_{I,H}$ is too cautious and the optimal response should be stronger. Along these lines, $X_{I,H}$ can intuitively be interpreted as the lower bound on X_U. By contrast, plugging in $X_{I,L} = X_U$, we are left with the second part of the FOC. In this part, the reaction to S_i implied by $X_{I,L}$ is too strong and should have been more modest. Hence, $X_{I,L}$ intuitively constitutes an upper bound on X_U. Facing these two relationships, the truth about the optimal X_U has to be situated somewhere in between $X_{I,H}$ and $X_{I,L}$. Compared with the first setting, in which it is optimal for the agent to stick to one strategy, here the optimal solution to the problem is to choose a middle strategy. However, the strategy cannot be pinned down analytically in terms of the parameters of the model.

5.3 Equilibrium price

After having guaranteed a solution to the FOC, thus the existence of optimal demand and describing its main features, the next step is to characterize the equilibrium price. We start by proving the existence of an equilibrium price that clears the market. Subsequently, we analyze and evaluate the characteristics of the equilibrium price trying to detect systematic price patterns. The market-clearing condition 1 requires that the demand of the uninformed investor equals noise trader's demand less the demand of the informed investors. This enables us to define uninformed demand as a function of informed and noise trader demand, $X_U(X_I, u)$. Notice that informed demand is a function of the signal and price, $X_I(S_i, P_{\lambda,i})$.

$$X_U(P_{\lambda,i}, S_i, u) = \left[\frac{u}{1-\lambda} - \frac{\lambda}{1-\lambda}\frac{E[\theta|S_i] - RP_{\lambda,i}}{\alpha Var[\theta|S_i]}\right] \tag{61}$$

Furthermore, we define the uninformed agents' utility in the respective regimes as a function of S_i and $X_U(P_{\lambda,i,u})$ with

$$\begin{aligned}
\Lambda_L(S_i, X_U(P_{\lambda,i})) &= -\alpha\left(RW_{0U} + (E[\theta|S_L] - RP)X_U(P_{\lambda,i}) - \frac{\alpha X_U(P_{\lambda,i})^2}{2}Var[\theta|S_L]\right), \\
\Lambda_H(S_i, X_U(P_{\lambda,i})) &= -\alpha\left(RW_{0U} + (E[\theta|S_H] - RP)X_U(P_{\lambda,i}) - \frac{\alpha X_U(P_{\lambda,i})^2}{2}Var[\theta|S_H]\right).
\end{aligned} \tag{62}$$

Substituting expressions 61 and 62 into the FOC produces an expression that implicitly characterizes the equilibrium price for each signal regime. Given $S_i = S_H$, this yields

$$\begin{aligned}0 = \omega_L \cdot \left(\alpha\left((E[\theta|S_L] - RP) - \alpha Var[\theta|S_L]\left[\frac{u}{1-\lambda} - \frac{\lambda}{1-\lambda}\frac{E[\theta|S_H] - RP}{\alpha Var[\theta|S_H]}\right]\right)\right) exp\left(\Lambda_L(S_H, P_{\lambda,H})\right), \\ +\omega_H \cdot \left(\alpha\left((E[\theta|S_H] - RP) - \alpha Var[\theta|S_H]\left[\frac{u}{1-\lambda} - \frac{\lambda}{1-\lambda}\frac{E[\theta|S_H] - RP}{\alpha Var[\theta|S_H]}\right]\right)\right) exp\left(\Lambda_H(S_H, P_{\lambda,H})\right).\end{aligned} \tag{63}$$

Proposition 2.

1. *There exists an equilibrium price function for each signal regime that fulfills condition 1 and is implicitly defined as*
   - *if* $S_i = S_H$

$$\begin{aligned}P_{\lambda,H} = \omega_L \frac{\left((1-\lambda)E[\theta|S_L] + \frac{\lambda Var[\theta|S_L]}{Var[\theta|S_H]}(E[\theta|S_H] - \alpha u Var[\theta|S_L]\right) \cdot exp\left(\Lambda_L(S_H, P_{\lambda,H})\right)}{\left(\omega_L \frac{Var[\theta|S_H] - \lambda(Var[\theta|S_H] - Var[\theta|S_L])}{Var[\theta|S_H]} \cdot exp\left(\Lambda_L(S_H, P_{\lambda,H})\right) + \omega_H \cdot exp\left(\Lambda_H(S_H, P_{\lambda,H})\right)\right)} \\ + \omega_H \frac{(E[\theta|S_H] - \alpha u Var[\theta|S_H]) \cdot exp\left(\Lambda_H(S_H, P_{\lambda,H})\right)}{\left(\omega_L \frac{Var[\theta|S_H] - \lambda(Var[\theta|S_H] - Var[\theta|S_L])}{Var[\theta|S_H]} \cdot exp\left(\Lambda_L(S_H, P_{\lambda,H})\right) + \omega_H \cdot exp\left(\Lambda_H(S_H, P_{\lambda,H})\right)\right)}\end{aligned} \tag{64}$$

   - *if* $S_i = S_L$

$$\begin{aligned}P_{\lambda,L} = \omega_L \frac{(E[\theta|S_L] - \alpha u Var[\theta|S_L]) \cdot exp\left(\Lambda_L(S_L, P_{\lambda,L})\right)}{\left(\omega_H \frac{Var[\theta|S_L] + \lambda(Var[\theta|S_H] - Var[\theta|S_L])}{Var[\theta|S_L]} \cdot exp\left(\Lambda_H(S_L, P_{\lambda,L})\right) + \omega_L \cdot exp\left(\Lambda_L(S_L, P_{\lambda,L})\right)\right)} \\ + \omega_H \frac{\left((1-\lambda)E[\theta|S_H] + \frac{\lambda Var[\theta|S_H]}{Var[\theta|S_L]}(E[\theta|S_L] - \alpha u Var[\theta|S_H]\right) \cdot exp\left(\Lambda_h(S_l, P_{\lambda,l})\right)}{\left(\omega_H \frac{Var[\theta|S_L] + \lambda(Var[\theta|S_H] - Var[\theta|S_L])}{Var[\theta|S_L]} \cdot exp\left(\Lambda_H(S_L, P_{\lambda,L})\right) + \omega_L \cdot exp\left(\Lambda_L(S_L, P_{\lambda,L})\right)\right)}\end{aligned} \tag{65}$$

2. *The boundaries for the equilibrium prices of the economy,* $P_{\lambda,S_i} \in (\underline{P}, \bar{P})$, *are given by the full information prices of the economy.*
   - *If* $S_i > \hat{\theta}$, $\underline{P} = P_{\hat{\lambda},H}$ *and* $\bar{P} = P_{\hat{\lambda},L}$ *for all* $S > \hat{\theta}$
   - *If* $S_i < \hat{\theta}$, $\underline{P} = P_{\hat{\lambda},L}$ *and* $\bar{P} = P_{\hat{\lambda},H}$ *for all* $S < \hat{\theta}$

Proof. The proof of existence proceeds very similar to the proof of proposition 1 applying an intermediate value theorem argument. We show that the function specified in 63 is continuous and crosses zero at least once. The price functions stated in the first part of proposition 2 are derived by simplifying 63 and collecting all $P_{\lambda,i}$ that are not part of the exponential function on one side. The second part of proposition 2 is obtained by plugging in the full information prices into the implicit price function and evaluating the function for each signal regime. The function implicitly characterizing the equilibrium demand is again the derivative of a weighted sum of two moment-generating functions and hence continuous. □

Together, propositions 1 and 2 guarantee the existence of a noisy REE and describe its most important characteristics. Looking at the expression that indirectly defines uninformed demand and knowing from proposition 1 that the optimal demand of the uninformed is unique given S and independent of the signal regime H or L equation 61 elicits that this does not hold for price. If demand is unique and not signal-regime dependent, then the market-clearing equilibrium price has to be dependent on not only the signal but also the signal regime. Two potential prices support equation 61 holding constant the values of u and S_i, which only depend on the kind of signal $i \in \{H, L\}$. This dichotomy of the market-clearing equilibrium price is at the center of the model. The demand of the uninformed induces two different prices depending on the signal regime. Furthermore, we show in proposition 2 that the market price is bounded by the full information prices but never equals them. Combining these two facts, by definition there have to be two different price paths whenever the price in the economy converges to the full information price.

5.4 Price dynamics

In order to make a statement about the price dynamics, it is again necessary to look at more than one period. We assume an extension of the model towards an additional trading period, in which the uncertainty of the signal regime is resolved. The setting is completely analogous to that described in paragraph 4 with the hitherto-known alteration that the uninformed observe the realization of the signal. In the second period, after the signal type is revealed there is no uncertainty left in the economy and the full information price materializes. This gives rise to the price dynamics described in the next proposition.

Proposition 3.

- *If $S_i = S_L$, the price difference ΔP_L will always satisfy $\Delta P > 0$ for all $S_L > \hat{\theta}$ and $\Delta P < 0$ for all $S_L < \hat{\theta}$. In such a case, the economy will experience under-reaction*
- *If $S_i = S_H$, the price difference ΔP_H will always satisfy $\Delta P < 0$ for all $S_H > \hat{\theta}$ and $\Delta P > 0$ for all $S_H < \hat{\theta}$. In such a case, the economy will experience over-reaction*

Proof. Proposition 3 follows immediately from the second part of proposition 2. □

According to proposition 3, the observable price pattern is purely driven by the signal regime. Notice that the uninformed agent is unable to distinguish the signal regimes. As soon as he observes the signal, he does not have to decide in advance which strategy to play, but according to proposition 1 there exists an optimal demand for each signal independent of its type. This demand never resembles the demand of the informed agent but is either above or below it. In comparison to the first part of the paper, when the uninformed has to choose his strategy a priori, his demand is either too high or too low in only one state of the economy. The deviation—either over- or under-reaction to a specific signal regime—depends on the strategy of the uninformed and hence the parametrization of the economy. It is not possible to generate both over- and under-reaction given a specific calibration of parameters without introducing variation in mixture weights p. By contrast, both patterns are feasible in this economy and are solely dependent on the signal type. They occur as soon as the variances of the two signals differ. Neither the amount of informed agents λ or the amount of noise trading u nor the variance of the pay off of the risky asset σ_θ or the mixture weights have an influence on the existence of these price pattern. In the first setting, the combination of these parameter values decided whether the economy experiences under- or over-reaction, while in the extended model this is no longer true. Here, the cited parameters are simply able to influence the severity of the movement. Prices always exhibit over-reaction following the high-variance signal and under-reaction following the low-variance signal. Hence, both varieties are possible in general, albeit are not equally likely.

Overall, proposition 3 states that given the low-variance regime the equilibrium price will always be below the full information price, inducing a positive correlated price drift between the two trading periods. The opposite is true for the high signal regime. The intuition follows the idea that for the uninformed agents some weighted average of the demand of the informed agents should be optimal.

6 Conclusion

We propose a rather simple but intuitively tractable model alteration of the classical noisy rational expectations model that supports price patterns of over- and under-reaction without deviating from the assumption of rationality. By expanding the basic Grossman and Stiglitz (1980) setting and introducing a heterogeneous signal structure, we are able to explain empirical price anomalies with the help of a noisy rational expectations model. Accordingly, we do not rely on sophisticated mechanisms of information transmission or the like, but rather solely work with different levels of information characteristics. The idea is to enrich the information structure beyond the point that one part of the agents in the economy observes a certain signal and the reminder does not. Instead, we challenge the idea that information is homogeneous and allow for a second dimension comprising heterogeneity in in the information structure.

Our results show that financial markets systematically over- or under-react to different types of signals, giving rise to certain price patterns that could not be avoided ex ante. Furthermore, we show that the added layer of information on its own suffices to incur a systematic deviation from the classical price building mechanism in noisy REE models. The price system is unable to dissolve the information asymmetry in the economy in such a setting.

The implemented information structure splits up the value of possessing information into two distinct parts: first, the classical notion of information by introducing two noisy signals, whose value is observed by a group of agents; and second, the ability to evaluate the received information appropriately. We show that being able to judge the actual quality of one's information appropriately is a valuable asset. Simply receiving information is insufficient to make optimal inference.

Additionally, the results in the first part of the paper support the notion that it is always beneficial to participate in the market and use the information at hand, even though this behavior implicates getting things not exactly right. This "second-best" strategy of drawing inference is always superior to simply ignoring information at hand and acting independent of one's information set. Put briefly, according to our result acting on information not optimally in a "first-best" kind of way can be rational, while not utilizing information is definitely at odds with the notion of rationality.

Taking a look at the empirical literature, our implications suit many findings on price over- and under-reactions on financial markets. Furthermore, they do not contradict the vast amount of behavioral biases that are said to be at the heart of price dynamics such as over- and under-reaction. We rather view these biases as complementary to our model. The main contribution is to highlight that over- and under-reaction also occur if agents are fully rational without drawing on sophisticated mechanisms of information dissemination or the existence of future markets and the like. However, an important question that has not yet been answered is how the altered information structure affects the equilibrium in the information market. This issue has to be investigated in further detail as it is vital for establishing the value of information and its implications on the information-generating process on financial markets.

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Appendices

A Derivations and proofs

A.1 Distribution of adjusted volume

In detail, adjusted volume writes:

$$\nu_H = \hat{\theta} + \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_{\epsilon_H}^2}(S_H-\hat{\theta}) - \frac{\alpha}{2\lambda}\frac{\sigma_\theta^2\sigma_{\epsilon_H}^2}{\sigma_\theta^2+\sigma_{\epsilon_H}^2}(u-\hat{u})$$
$$\nu_L = \hat{\theta} + \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_{\epsilon_L}^2}(S_L-\hat{\theta}) - \frac{\alpha}{2\lambda}\frac{\sigma_\theta^2\sigma_{\epsilon_L}^2}{\sigma_\theta^2+\sigma_{\epsilon_L}^2}(u-\hat{u})$$

which can be rewritten as

$$\nu_H = \hat{\theta} + F_H(S_H-\hat{\theta} - \underbrace{\frac{\alpha\sigma_{\epsilon_H}^2}{2\lambda}}_{K_H}(u-\hat{u}))$$
$$\nu_L = \hat{\theta} + F_L(S_L-\hat{\theta} - \underbrace{\frac{\alpha\sigma_{\epsilon_L}^2}{2\lambda}}_{K_L}(u-\hat{u}))$$

Adjusted volume ν is distributed as a Gaussian mixture with mixture weights p and $1-p$ inherited from 3 and components given by

$$\nu_H \sim \mathcal{N}\left(\hat{\theta}, {F_H}^2(\sigma_\theta^2+\sigma_{\epsilon_H}^2+(\frac{\alpha\sigma_{\epsilon_H}^2}{2\lambda})^2\sigma_u^2)\right) \quad and \quad \nu_L \sim \mathcal{N}\left(\hat{\theta}, {F_L}^2(\sigma_\theta^2+\sigma_{\epsilon_L}^2+(\frac{\alpha\sigma_{\epsilon_L}^2}{2\lambda})^2\sigma_u^2)\right)$$

Given the distributions of ν_H and ν_L, adjusted volume ν_i and θ for each mixture component are distributed jointly normal.

$$\begin{matrix}\theta\\ \nu_H\end{matrix} \sim \mathcal{N}\left[\begin{pmatrix}\hat{\theta}\\ \hat{\theta}\end{pmatrix}, \begin{pmatrix}\sigma_\theta^2 & F_H\sigma_\theta^2\\ F_H\sigma_\theta^2 & F_H^2(\sigma_\theta^2+\sigma_{\epsilon_H}^2+K_H^2\sigma_u^2)\end{pmatrix}\right]$$
$$\begin{matrix}\theta\\ \nu_L\end{matrix} \sim \mathcal{N}\left[\begin{pmatrix}\hat{\theta}\\ \hat{\theta}\end{pmatrix}, \begin{pmatrix}\sigma_\theta^2 & F_L\sigma_\theta^2\\ F_L\sigma_\theta^2 & F_L^2(\sigma_\theta^2+\sigma_{\epsilon_L}^2+K_L^2\sigma_u^2)\end{pmatrix}\right]$$

By the projection theorem, the expected value of θ given ν_i is

$$E[\theta|\nu_H] = \hat{\theta} + \frac{1}{F_H}\frac{\sigma_\theta^2}{(\sigma_\theta^2+\sigma_{\epsilon_H}^2+K_H^2\sigma_u^2)}(\nu_H-\hat{\theta}) = \hat{\theta} + \underbrace{\frac{\sigma_\theta^2+\sigma_{\epsilon_H}^2}{(\sigma_\theta^2+\sigma_{\epsilon_H}^2+K_H^2\sigma_u^2)}}_{G_H}(\nu_H-\hat{\theta})$$

$$E[\theta|\nu_L] = \hat{\theta} + \frac{1}{F_L}\frac{\sigma_\theta^2}{(\sigma_\theta^2+\sigma_{\epsilon_L}^2+K_L^2\sigma_u^2)}(\nu_L-\hat{\theta}) = \hat{\theta} + \underbrace{\frac{\sigma_\theta^2+\sigma_{\epsilon_L}^2}{(\sigma_\theta^2+\sigma_{\epsilon_L}^2+K_L^2\sigma_u^2)}}_{G_L}(\nu_L-\hat{\theta})$$

the variance of θ given ν_i is

$$
\begin{aligned}
Var[\theta|\nu_H] &= \sigma_\theta^2 - \frac{(F_H\sigma_\theta^2)^2}{{F_H}^2(\sigma_\theta^2+\sigma_{\epsilon_H}^2+{K_H}^2\sigma_u^2)} = \frac{\sigma_\theta^2(\sigma_{\epsilon_H}^2+{K_H}^2\sigma_u^2)}{(\sigma_\theta^2+\sigma_{\epsilon_H}^2+{K_H}^2\sigma_u^2)} \\
Var[\theta|\nu_L] &= \sigma_\theta^2 - \frac{(F_L\sigma_\theta^2)^2}{{F_L}^2(\sigma_\theta^2+\sigma_{\epsilon_L}^2+{K_L}^2\sigma_u^2)} = \frac{\sigma_\theta^2(\sigma_{\epsilon_L}^2+{K_L}^2\sigma_u^2)}{(\sigma_\theta^2+\sigma_{\epsilon_L}^2+{K_L}^2\sigma_u^2)}
\end{aligned} \tag{66}
$$

A.2 Utility of the informed

Utility of the informed is given by

$$
E[V(W_{1I})|S_i] = -exp\Big[-\alpha\big(E[W_{1I}|S_i] - \frac{\alpha}{2}Var[W_{1I}|S_i]\big)\Big] \tag{67}
$$

with W_{1I} being

$$
W_{1I} = RW_{0I} + (\theta - RP)X_I,
$$

Using 39 and plugging in W_{1I} and X_I

$$
\begin{aligned}
E[W_{1I}|S_i] &= E\Big[RW_{0I} + (\theta - RP_{\lambda;i,j})X_I\Big|S_i\Big] \\
&= E\Big[RW_{0I} + (\theta - RP_{\lambda;i,j})\frac{E[\theta|S_i]-RP_{\lambda;i,j}}{\alpha * Var[\theta|S_i]}|S_i\Big] \\
&= RW_{0I} + \frac{(E[\theta|S_i]-RP_{\lambda;i,j})^2}{\alpha * Var[\theta|S_i]}
\end{aligned}
$$

$$
\begin{aligned}
Var[W_{1I}|S_i] &= Var\Big[RW_{0I} + (\theta - RP_{\lambda;i,j})X_I\Big|S_i\Big] \\
&= Var\Big[(\theta - RP_{\lambda;i,j})\frac{E[\theta|S_i]-RP_{\lambda;i,j}}{\alpha * Var[\theta|S_i]}|S_i\Big] \\
&= \frac{(E[\theta|S_i]-RP_{\lambda;i,j})^2}{\alpha^2 * Var[\theta|S_i]^2}Var\Big[(\theta - RP)\Big|S_i\Big] \\
&= \frac{(E[\theta|S_i]-RP_{\lambda;i,j})^2}{\alpha^2 * Var[\theta|S_i]^2}Var[\theta|S_i] \\
&= \frac{(E[\theta|S_i]-RP_{\lambda;i,j})^2}{\alpha^2 * Var[\theta|S_i]}
\end{aligned}
$$

one gets

$$
\begin{aligned}
E[V(W_{1I})|S_i] &= -exp\Big[-\alpha\Big(RW_{0I} + \frac{(E[\theta|S_i]-RP_{\lambda;i,j})^2}{\alpha * Var[\theta|S_i]} - \frac{\alpha}{2}\frac{(E[\theta|S_i]-RP_{\lambda;i,j})^2}{\alpha^2 * Var[\theta|S_i]}\Big)\Big] \\
&= -exp\Big[-\alpha\Big(RW_{0I} + \frac{1}{2}\frac{(E[\theta|S_i]-RP_{\lambda;i,j})^2}{\alpha * Var[\theta|S_i]}\Big)\Big] \\
&= -exp\Big[-\alpha RW_{0I} - \frac{1}{2}\frac{(E[\theta|S_i]-RP_{\lambda;i,j})^2}{Var[\theta|S_i]}\Big] \\
&= -exp\Big[-\alpha RW_{0I}\Big] * exp\Big[-\frac{1}{2}\frac{(E[\theta|S_i]-RP_{\lambda;i,j})^2}{Var[\theta|S_i]}\Big].
\end{aligned} \tag{68}
$$

We proceed by evaluating the second component of 68 conditional on ν_i

$$E\left[exp\Big(-\frac{1}{2}\frac{(E[\theta|S_i]-RP_{\lambda;i,j})^2}{Var[\theta|S_i]}\Big)\Big|\nu_i\right]. \tag{69}$$

In order to determine the above expression, it is important to establish some intermediate results. By the law of iterated expectations

$$\begin{aligned} E[\theta|S_i] &= \bar{\theta} + \frac{cov[\theta S_i]}{Var[S_i]}(S_i - \hat{\theta}) \\ E\big[E[\theta|S_i]\big|\nu_i\big] &= E[\theta|\nu_i] \end{aligned} \tag{70}$$

Notice the fact that the only stochastic variables are S_i and u. Thus, the variance of $E[\theta|S_i]$ given ν_i, according to the projection theorem writes

$$Var\big[E[\theta|S_i]\big|\nu_i\big] = F_i^2 Var[S_i] - \frac{\big(F_i^2 Var[S_i]\big)^2}{Var[\nu_i]} = \frac{F_i\sigma_\theta^2 * K_i{}^2\sigma_u^2}{(\sigma_\theta^2+\sigma_{\epsilon_i}^2+K_i{}^2\sigma_u^2)} \tag{71}$$

With this in mind, we define a new variable Z as

$$Z = \frac{(E[\theta|S_i]-RP_{\lambda;i,j})}{\sqrt{Var\big[E[\theta|S_i]\big|\nu_i\big]}} \tag{72}$$

and plug it into expression 69.

$$E\left[exp\Big(-\frac{1}{2}\frac{(E[\theta|S_i]-RP_{\lambda;i,j})^2}{Var[\theta|S_i]}\Big)\Big|\nu_i\right] = E\left[exp\left(-\frac{Var\left[E[\theta|S_i]\big|\nu_i\right]}{2Var\left[\theta|S_i\right]}Z^2\right)\Bigg|\nu_i\right]. \tag{73}$$

In the following we closely follow the lines of Grossman and Stiglitz (1980) Appendix B. Knowing that S_i is normal for S_H and S_L, $E[\theta|S_i]$ is also normal. The same holds for the distribution of $E[\theta|S_i]$ conditional on ν_i, making Z^2 the square of a standardized normal variable with some mean μ and variance of 1 and hence distributed non-central chi-square with one df. (see Rao, 1973, p. 173). To solve 73 we use the moment-generating function of the non-central chi-square distribution , which in our case is given by

$$E[e^{tZ^2}|\nu_i] = \frac{1}{\sqrt{1-2t}}exp\left[\frac{E([Z|\nu_i])^2 t}{1-2t}\right] \tag{74}$$

with $t = -\frac{Var[E[\theta|S_i]|\nu_i]}{2Var[\theta|S_i]}$ and the non-centrality parameter $\frac{(E\left[E[\theta|S_i]|\nu_i\right]-RP_{\lambda;i,j})^2}{Var\left[E[\theta|S_i]|\nu_i\right]}$. Substituting

$$1-2t = 1 + \frac{Var\big[E[\theta|S_i]\big|\nu_i\big]}{Var[\theta|S_i]} = \frac{Var\big[E[\theta|S_i]\big|\nu_i\big]+Var[\theta|S_i]}{Var[\theta|S_i]} = \frac{Var[\theta|\nu_i]}{Var[\theta|S_i]} \tag{75}$$

and

$$\frac{t}{1-2t} = \frac{Var[\theta|S_i]}{Var[\theta|\nu_i]}\Big(-\frac{Var\big[E[\theta|S_i]\big|\nu_i\big]}{2Var[\theta|S_i]}\Big) = -\frac{Var\big[E[\theta|S_i]\big|\nu_i\big]}{2Var[\theta|\nu_i]} \tag{76}$$

into 74 yields

$$\begin{aligned} &E\Big[exp\Big(-\frac{Var\big[E[\theta|S_i]\big|\nu_i\big]}{2Var[\theta|S_i]}Z^2|\nu_i\Big)\Big] \\ &= \sqrt{\frac{Var[\theta|S_i]}{Var[\theta|\nu_i]}}exp\Big[-\frac{Var\big[E[\theta|S_i]\big|\nu_i\big]}{2Var[\theta|\nu_i]}\frac{\Big(E\big[E[\theta|S_i]\big|\nu_i\big]-RP_{\lambda;i,j}\Big)^2}{Var\big[E[\theta|S_i]\big|\nu_i\big]}\Big] \\ &= \sqrt{\frac{Var[\theta|S_i]}{Var[\theta|\nu_i]}}exp\Big[-\frac{\Big(E[\theta|\nu_i]-RP_{\lambda;i,j}\Big)^2}{2Var[\theta|\nu_i]}\Big] \end{aligned} \tag{77}$$

Using the result of 77 and plugging in 68 yields the expected utility of the informed agent given the ν_i

$$E\left[E[V(W_{1I})|S_i]|\nu_i\right] = V(RW_{0I})\sqrt{\frac{Var[\theta|S_i]}{Var[\theta|\nu_i]}} * exp\Big[-\frac{\Big(E[\theta|\nu_i]-RP_{\lambda;i,j}\Big)^2}{2Var[\theta|\nu_i]}\Big]. \tag{78}$$

A.3 Utility of the uninformed

Starting at

$$E[V(W_{1U})|\nu_i] = -exp\Big[-\alpha\big(E[W_{1U}|\nu_i]-\frac{\alpha}{2}Var[W_{1U}|\nu_i]\big)\Big] \tag{79}$$

with W_{1U} being

$$W_{1U} = RW_{0U} + (\theta - RP)X_U. \tag{80}$$

Using 79 and plugging in W_{1U} and X_U

$$\begin{aligned} E[W_{1U}|\nu_i] &= E\Big[RW_{0U} + (\theta - RP_{\lambda;i,j})X_U\Big|\nu_i\Big] \\ &= E\Big[RW_{0U} + (\theta - RP_{\lambda;i,j})\frac{E[\theta|\nu_i^j, P_{\lambda;i,j}] - RP_{\lambda;i,j}}{\alpha * Var[\theta|\nu_i^j, P_{\lambda;i,j}]}|\nu_i\Big] \\ &= RW_{0U} + E\Big[(\theta - RP_{\lambda;i,j})\frac{E[\theta|\nu_i^j, P_{\lambda;i,j}] - RP_{\lambda;i,j}}{\alpha * Var[\theta|\nu_i^j, P_{\lambda;i,j}]}|\nu_i\Big] \end{aligned} \tag{81}$$

$$
\begin{aligned}
Var[W_{1U}|\nu_i] &= Var\Big[RW_{0U} + (\theta - RP_{\lambda;i,j})X_U\Big|\nu_i\Big] \\
&= Var\Big[\frac{E[\theta|\nu_i^j, P_{\lambda;i,j}] - RP_{\lambda;i,j}}{\alpha * Var[\theta|\nu_i^j, P_{\lambda;i,j}]}(\theta - RP_{\lambda;i,j})\Big|\nu_i\Big] \\
&= \left(\frac{E[\theta|\nu_i^j, P_{\lambda;i,j}] - RP_{\lambda;i,j}}{\alpha * Var[\theta|\nu_i^j, P_{\lambda;i,j}]}\right)^2 Var\Big[(\theta - RP_{\lambda;i,j})\Big|\nu_i\Big] \\
&= \frac{\Big(E[\theta|\nu_i^j, P_{\lambda;i,j}] - RP_{\lambda;i,j}\Big)^2}{\Big(\alpha * Var[\theta|\nu_i^j, P_{\lambda;i,j}]\Big)^2} Var\left[\theta|\nu_i\right] \\
&= \frac{\Big(E[\theta|\nu_i^j, P_{\lambda;i,j}] - RP_{\lambda;i,j}\Big)^2}{\alpha * Var[\theta|\nu_i^j, P_{\lambda;i,j}]} \frac{Var\left[\theta|\nu_i\right]}{\alpha * Var[\theta|\nu_i^j, P_{\lambda;i,j}]}
\end{aligned} \tag{82}
$$

one gets

$$
\begin{aligned}
E[V(W_{1U})|\nu_i] =& - exp\Big[- \alpha\Big(RW_{0U} + E\Big[(\theta - RP_{\lambda;i,j})\frac{E[\theta|\nu_i^j, P_{\lambda;i,j}] - RP_{\lambda;i,j}}{\alpha * Var[\theta|\nu_i^j, P_{\lambda;i,j}]}\Big|\nu_i\Big] \\
&- \frac{\alpha}{2}\frac{\Big(E[\theta|\nu_i^j, P_{\lambda;i,j}] - RP_{\lambda;i,j}\Big)^2}{\alpha * Var[\theta|\nu_i^j, P_{\lambda;i,j}]}\frac{Var\left[\theta|\nu_i\right]}{\alpha * Var[\theta|\nu_i^j, P_{\lambda;i,j}]}\Big)\Big] \\
=& -exp\Big[- \alpha RW_{0U} - \frac{E[\theta|\nu_i^j, P_{\lambda;i,j}] - RP_{\lambda;i,j}}{\alpha * Var[\theta|\nu_i^j, P_{\lambda;i,j}]} \\
&\left((E[\theta|\nu_i] - RP_{\lambda;i,j}) - \frac{1}{2}\left(E[\theta|\nu_i^j, P_{\lambda;i,j}] - RP_{\lambda;i,j}\right)\frac{Var\left[\theta|\nu_i\right]}{Var[\theta|\nu_i^j, P_{\lambda;i,j}]}\right)\Big].
\end{aligned} \tag{83}
$$

After factoring expected utility of the uninformed writes

$$
\begin{aligned}
&E\left[E[V(W_{1U})|\nu_i]\right] = -exp\left[-\alpha RW_{0U}\right] * \\
&E\left[exp\left[-\frac{E[\theta|\nu_i^j, P_{\lambda;i,j}] - RP_{\lambda;i,j}}{\alpha * Var[\theta|\nu_i^j, P_{\lambda;i,j}]}\left((E[\theta|\nu_i] - RP_{\lambda;i,j}) - \frac{1}{2}\left(E[\theta|\nu_i^j, P_{\lambda;i,j}] - RP_{\lambda;i,j}\right)\frac{Var\left[\theta|\nu_i\right]}{Var[\theta|\nu_i^j, P_{\lambda;i,j}]}\right)\right]\right]
\end{aligned} \tag{84}
$$

If $i = j$, 84 reduces to

$$
\begin{aligned}
E[V(W_{1U})|\nu_i] &= -exp\Big[- \alpha RW_{0U}\Big] * exp\Big[- \frac{1}{2}\frac{(E[\theta|\nu_i] - RP_{\lambda;i})^2}{Var[\theta|\nu_i]}\Big] \\
&= V(RW_{0U}) * exp\Big[- \frac{1}{2}\frac{(E[\theta|\nu_i] - RP_{\lambda;i})^2}{Var[\theta|\nu_i]}\Big].
\end{aligned} \tag{85}
$$

A.4 Proof of Theorem 3.1

Lemma A.1. *The utility of the uniformed agents after entering the market always exceeds the end of period utility of the initial wealth of the uninformed agents. Therefore, staying out of the market is not a feasible strategy. Furthermore, it is always superior for the uninformed agents to play strategy (H) compared to completely staying out of the market or entering the market* *without trying to infer information from the price system.*

Not trying to infer information from the price system can be thought of as maximizing based on an unconditional information set.

Proof. For the proof, we simply consider the low uncertainty regime, as it is the scenario when playing strategy (H) is not by definition superior to the other strategies. If the uninformed agent gets his inference right, which is in the scenario of the high volatility regime, the expected utility of the uninformed using his information set in the right way by definition exceeds the utility of the uninformed if ignored the information that he receives from price. We proceed by defining the expected utility of the uninformed for the three scenarios: (i) not entering the market at all; (ii) entering given unconditional expectations; and (iii) playing strategy (H). Subsequently, we show that playing strategy (H) always strictly dominates the other two options.

The utility if the uninformed stays away from the market is given by

$$E[V(W_{0U})] = -exp\left[-\alpha W_{0U}\right] \tag{86}$$

The utility of the uninformed when maximizing given unconditional expectations is calculated as follows. According to equation 25 the demand of the uninformed writes

$$X_U = \frac{E[\theta] - P_{\lambda,H}}{\alpha * Var[\theta]}, \tag{87}$$

and the respective price according to 30 would be given by

$$P_{\lambda,L,Uc} = \frac{\dfrac{\lambda}{\alpha Var[\theta|S_L]}\nu_L + \dfrac{(1-\lambda)}{\alpha Var[\theta]}E[\theta]}{\dfrac{\lambda}{\alpha Var[\theta|S_L]} + \dfrac{(1-\lambda)}{\alpha Var[\theta]}}. \tag{88}$$

The expected value of wealth of the uninformed conditional on the true adjusted volume is given by plugging 87 into 80 and taking expectations

$$E[W_{1U}|\nu_L] = W_{0U} + (E[\theta|\nu_L] - P_{\lambda;L,Uc})\frac{E[\theta] - P_{\lambda;L,Uc}}{\alpha * Var[\theta]}, \tag{89}$$

while the expected variance of wealth of the uninformed conditional on the true adjusted volume is given by plugging 87 into 80 and determining the variance which writes

$$Var[W_{1U}|\nu_L] = \frac{(E[\theta] - P_{\lambda;L,Uc})^2}{\alpha * Var[\theta]}\frac{Var[\theta|\nu_L]}{\alpha * Var[\theta]}. \tag{90}$$

Plugging 89 and 82 into 46 and writing everything in terms of the parameters of the model and as a function of $(\nu_L - \hat{\theta})$ yields

$$E\left[E[V(W_{1U})|\nu_i]\right] = -exp\left[-\alpha RW_{0U}\right] * \\ E\left[exp\left[-\frac{\lambda^2\sigma^2_{\epsilon_L}\left((\lambda-2)\lambda+\alpha^2\sigma^2_{\epsilon_L}\sigma^2_u\right)(\sigma^2_{\epsilon_L}+\sigma^2_\theta)}{2\sigma^2_\theta(\sigma^2_{\epsilon_L}+\lambda\sigma^2_\theta)^2(\lambda^2\sigma^2_{\epsilon_L}+\alpha^2\sigma^4_{\epsilon_L}\sigma^2_u+\lambda^2\sigma^2_\theta)}(\nu_L-\hat{\theta})^2\right]\right]. \tag{91}$$

Now we define a new variable Z_U

$$Z_U = \frac{(\nu_L - \hat{\theta})}{\sqrt{Var[\nu_L]}}. \tag{92}$$

Since ν_L is normal with Mean $\hat{\theta}$, Z_U is distributed standard normal and hence Z_U^2 is distributed central chi-square with one degree of freedom (see Rao, 1973, p. 173). Plugging Z_U^2 into 91 yields

$$E\left[E[V(W_{1U})|\nu_i]\right] = -exp\left[-\alpha RW_{0U}\right] * \\ E\left[exp\left[-\frac{\lambda^2\sigma^2_{\epsilon_L}\left((\lambda-2)\lambda+\alpha^2\sigma^2_{\epsilon_L}\sigma^2_u\right)(\sigma^2_{\epsilon_L}+\sigma^2_\theta)}{2\sigma^2_\theta(\sigma^2_{\epsilon_L}+\lambda\sigma^2_\theta)^2(\lambda^2\sigma^2_{\epsilon_L}+\alpha^2\sigma^4_{\epsilon_L}\sigma^2_u+\lambda^2\sigma^2_\theta)} * Var[\nu_L] * Z_U^2\right]\right]. \tag{93}$$

Given the distributional characteristics of Z_U^2, the expectation can be solved by using the moment-generating function of a central chi-square distribution, which is given by

$$E[e^{tZ^2}] = \frac{1}{\sqrt{1-2t}}, \tag{94}$$

with

$$t_{Uc} = -\frac{\lambda^2\sigma^2_{\epsilon_L}\left((\lambda-2)\lambda+\alpha^2\sigma^2_{\epsilon_L}\sigma^2_u\right)(\sigma^2_{\epsilon_L}+\sigma^2_\theta)}{2\sigma^2_\theta(\sigma^2_{\epsilon_L}+\lambda\sigma^2_\theta)^2(\lambda^2\sigma^2_{\epsilon_L}+\alpha^2\sigma^4_{\epsilon_L}\sigma^2_u+\lambda^2\sigma^2_\theta)} * Var[\nu_L]. \tag{95}$$

If the uninformed agent plays the conservative strategy (H), the parameters of the model are given as stated in the second part of section 3.3. The expected value and variance of end-of-period wealth are given by

$$E[W_{1U}|\nu_L^H] = RW_{0U} + \left(E\left[\theta|\nu_L\right] - RP_{\lambda;L,H}\right)\frac{E[\theta|\nu_L^H, P_{\lambda;L,H}] - RP_{\lambda;L,H}}{\alpha * Var[\theta|\nu_L^H, P_{\lambda;L,H}]}, \tag{96}$$

$$Var[W_{1U}|\nu_L^H] = \frac{\left(E[\theta|\nu_L^H, P_{\lambda;L,H}] - RP_{\lambda;L,H}\right)^2}{\alpha * Var[\theta|\nu_L^H, P_{\lambda;L,H}]}\frac{Var\left[\theta|\nu_L\right]}{\alpha * Var[\theta|\nu_L^H, P_{\lambda;L,H}]}. \tag{97}$$

Plugging 96 and 97 into 46 and writing everything in terms of the parameters of the model and as a function of $(\nu_L - \hat{\theta})$ yields

$$E\left[E[V(W_{1U})|\nu_i]\right] = -exp\left[-\alpha R W_{0U}\right] *$$
$$E\left[exp\left[-\frac{\alpha^4\lambda^2\sigma_{\epsilon_L}^2\sigma_{\epsilon_H}^2\sigma_u^4\left(2\lambda\sigma_{\epsilon_L}^2+\sigma_{\epsilon_H}^2\left((\lambda-2)\lambda+\alpha^2\sigma_{\epsilon_L}^2\sigma_u^2\right)\right)(\sigma_{\epsilon_L}^2+\sigma_\theta^2)^2}{2(\lambda^2\sigma_{\epsilon_L}^2+\alpha^2\sigma_{\epsilon_L}^4\sigma_u^2+\lambda^2\sigma_\theta^2)\left(\sigma_{\epsilon_L}^2\left(\lambda^2+\alpha^2\sigma_{\epsilon_H}^2\sigma_u^2\right)\sigma_\theta+\lambda\left(\lambda+\alpha^2\sigma_{\epsilon_H}^2\sigma_u^2\right)\sigma_\theta^3\right)^2} * (\nu_L-\hat{\theta})^2\right]\right]. \tag{98}$$

Now, one can again substitute Z_U and solve the expectation using the moment-generating function given by 94 with t defined as

$$t_{L,H} = -\frac{\alpha^4\lambda^2\sigma_{\epsilon_L}^2\sigma_{\epsilon_H}^2\sigma_u^4\left(2\lambda\sigma_{\epsilon_L}^2+\sigma_{\epsilon_H}^2\left((\lambda-2)\lambda+\alpha^2\sigma_{\epsilon_L}^2\sigma_u^2\right)\right)(\sigma_{\epsilon_L}^2+\sigma_\theta^2)^2}{2(\lambda^2\sigma_{\epsilon_L}^2+\alpha^2\sigma_{\epsilon_L}^4\sigma_u^2+\lambda^2\sigma_\theta^2)\left(\sigma_{\epsilon_L}^2\left(\lambda^2+\alpha^2\sigma_{\epsilon_H}^2\sigma_u^2\right)\sigma_\theta+\lambda\left(\lambda+\alpha^2\sigma_{\epsilon_H}^2\sigma_u^2\right)\sigma_\theta^3\right)^2} * Var[\nu_L]. \tag{99}$$

As utility is negative due to the CARA setting, the lower the term, the higher the utility of the uninformed agents. As the moment-generating function 94 is monotone decreasing with the absolute value of t, we show that it generally holds that $|t_{L,H}| > |t_{UC}$. After some tedious algebra, the problem boils down to

$$\frac{\alpha^4\sigma_{\epsilon_H}^2\sigma_u^4\left(\sigma_{\epsilon_H}^2\left((\lambda-2)\lambda+\alpha^2\sigma_{\epsilon_L}^2\sigma_u^2\right)+2\lambda\sigma_{\epsilon_L}^2\right)}{\left(\sigma_\theta\sigma_{\epsilon_L}^2\left(\alpha^2\sigma_{\epsilon_H}^2\sigma_u^2+\lambda^2\right)+\lambda\sigma_\theta^3\left(\alpha^2\sigma_{\epsilon_H}^2\sigma_u^2+\lambda\right)\right)^2} > \frac{(\lambda-2)\lambda+\alpha^2\sigma_{\epsilon_L}^2\sigma_u^2}{\sigma_\theta^2\left(\lambda\sigma_\theta^2+\sigma_{\epsilon_L}^2\right)^2} \tag{100}$$

which is true for all $\forall\ \sigma_\theta^2 \geq 1$, $\sigma_u^2 \geq 1$, $1 \leq \sigma_{\epsilon_L}^2 < \sigma_{\epsilon_H}^2$, $1 \leq \alpha$ and $0 < \lambda < 1$. □

Lemma A.2. *There exists a value p^* determined by the parameters of the model that marks the threshold $p*$, determining the overall optimality between strategies (H) and (L). Given $p < p^*$, it is always superior for the uninformed agents to play strategy (L) compared to strategy (H). Given $p > p^*$ it is always superior for the uninformed agents to play strategy (H) compared to strategy (L)*

Proof. If the uninformed agent plays strategy L and the true signal regime is given by H, the parameters and variables of the model are

$$X_{U;H,L} = \frac{E[\theta|\nu_H^L, P_{\lambda,HL}] - P_{\lambda,HL}}{\alpha * Var[\theta|\nu_H^L, P_{\lambda,HL}]}, \tag{101}$$

$$\nu_H^L = \nu_H + \frac{Var[\theta|S_L] - Var[\theta|S_H]}{Var[\theta|S_H]}(\nu_H - P_{\lambda,HL}), \tag{102}$$

$$P_{\lambda;H,L} = \frac{\frac{\lambda}{\alpha Var[\theta|S_H]}\nu_H + \frac{(1-\lambda)}{\alpha Var[\theta|\nu_L]}[\hat{\theta} + G_L(1 + \frac{Var[\theta|S_L] - Var[\theta|S_H]}{Var[\theta|S_H]})\nu_H - \hat{\theta})]}{\frac{\lambda}{\alpha Var[\theta|S_H]} + \frac{(1-\lambda)}{\alpha Var[\theta|\nu_L]}(1 + G_L(\frac{Var[\theta|S_L] - Var[\theta|S_H]}{Var[\theta|S_H]})}. \quad (103)$$

The expected value and variance of end-of-period wealth are given by

$$E[W_{1U}|\nu_H^L] = RW_{0U} + (E[\theta|\nu_H] - RP_{\lambda;H,L})\frac{E[\theta|\nu_H^L, P_{\lambda;H,L}] - RP_{\lambda;H,L}}{\alpha * Var[\theta|\nu_H^L, P_{\lambda;H,L}]}, \quad (104)$$

and

$$Var[W_{1U}|\nu_H^L] = \frac{\left(E[\theta|\nu_H^L, P_{\lambda;H,L}] - RP_{\lambda;H,L}\right)^2}{\alpha * Var[\theta|\nu_H^L, P_{\lambda;H,L}]} \frac{Var\left[\theta|\nu_H\right]}{\alpha * Var[\theta|\nu_H^L, P_{\lambda;H,L}]}. \quad (105)$$

Plugging 104 and 105 into 46 and writing everything in terms of the parameters of the model and as a function of $(\nu_L - \hat{\theta})$ yields

$$E\left[E[V(W_{1U})|\nu_H]\right] = -exp\left[-\alpha RW_{0U}\right] *$$
$$E\left[exp\left[-\frac{\alpha^4\lambda^2\sigma_{\epsilon_L}^2\sigma_{\epsilon_H}^2\sigma_u^4\left((\lambda-2)\lambda\sigma_{\epsilon_L}^2 + \sigma_{\epsilon_H}^2\left(2\lambda + \alpha^2\sigma_{\epsilon_L}^2\sigma_u^2\right)\right)(\sigma_{\epsilon_H}^2 + \sigma_\theta^2)^2}{2(\lambda^2\sigma_{\epsilon_H}^2 + \alpha^2\sigma_{\epsilon_H}^4\sigma_u^2 + \lambda^2\sigma_\theta^2)\left(\sigma_{\epsilon_H}^2\left(\lambda^2 + \alpha^2\sigma_{\epsilon_L}^2\sigma_u^2\right)\sigma_\theta + \lambda\left(\lambda + \alpha^2\sigma_{\epsilon_l}^2\sigma_u^2\right)\sigma_\theta^3\right)^2} * (\nu_H - \hat{\theta})^2\right]\right]. \quad (106)$$

Defining $Z_{U,H}$ as

$$Z_{U,H} = \frac{(\nu_H - \hat{\theta})}{\sqrt{Var[\nu_H]}}, \quad (107)$$

and plugging in yields

$$E\left[E[V(W_{1U})|\nu_H]\right] = -exp\left[-\alpha RW_{0U}\right] *$$
$$E\left[exp\left[-\frac{\alpha^4\lambda^2\sigma_{\epsilon_L}^2\sigma_{\epsilon_H}^2\sigma_u^4\left((\lambda-2)\lambda\sigma_{\epsilon_L}^2 + \sigma_{\epsilon_H}^2\left(2\lambda + \alpha^2\sigma_{\epsilon_L}^2\sigma_u^2\right)\right)(\sigma_{\epsilon_H}^2 + \sigma_\theta^2)^2}{2(\lambda^2\sigma_{\epsilon_H}^2 + \alpha^2\sigma_{\epsilon_H}^4\sigma_u^2 + \lambda^2\sigma_\theta^2)\left(\sigma_{\epsilon_H}^2\left(\lambda^2 + \alpha^2\sigma_{\epsilon_L}^2\sigma_u^2\right)\sigma_\theta + \lambda\left(\lambda + \alpha^2\sigma_{\epsilon_l}^2\sigma_u^2\right)\sigma_\theta^3\right)^2} * Var[\nu_H] * Z_{U,H}^2\right]\right]. \quad (108)$$

Using 94, t is given by

$$\begin{aligned} t_{H,L} &= -\frac{\alpha^4\lambda^2\sigma_{\epsilon_L}^2\sigma_{\epsilon_H}^2\sigma_u^4\left((\lambda-2)\lambda\sigma_{\epsilon_L}^2 + \sigma_{\epsilon_H}^2\left(2\lambda + \alpha^2\sigma_{\epsilon_L}^2\sigma_u^2\right)\right)(\sigma_{\epsilon_H}^2 + \sigma_\theta^2)^2}{2(\lambda^2\sigma_{\epsilon_H}^2 + \alpha^2\sigma_{\epsilon_H}^4\sigma_u^2 + \lambda^2\sigma_\theta^2)\left(\sigma_{\epsilon_H}^2\left(\lambda^2 + \alpha^2\sigma_{\epsilon_L}^2\sigma_u^2\right)\sigma_\theta + \lambda\left(\lambda + \alpha^2\sigma_{\epsilon_l}^2\sigma_u^2\right)\sigma_\theta^3\right)^2} * Var[\nu_H] \\ &= -\frac{\alpha^4\sigma_{\epsilon_L}^2\sigma_{\epsilon_H}^2\sigma_u^4\left((\lambda-2)\lambda\sigma_{\epsilon_L}^2 + \sigma_{\epsilon_H}^2\left(2\lambda + \alpha^2\sigma_{\epsilon_L}^2\sigma_u^2\right)\right)(\sigma_{\epsilon_H}^2 + \sigma_\theta^2)^2}{2\left(\sigma_{\epsilon_H}^2\left(\lambda^2 + \alpha^2\sigma_{\epsilon_L}^2\sigma_u^2\right)\sigma_\theta + \lambda\left(\lambda + \alpha^2\sigma_{\epsilon_l}^2\sigma_u^2\right)\sigma_\theta^3\right)^2} \end{aligned} \quad (109)$$

Remember that the utility in case the uninformed gets the signal regime right is given by 47 and writes

$$E[V(W_{1U})|\nu_L] = V(RW_{0U}) * exp\Big[-\frac{1}{2}\frac{(E[\theta|\nu_L] - RP_{\lambda;L})^2}{Var[\theta|\nu_L]}\Big]. \tag{110}$$

The corresponding price is given by

$$P_{\lambda,L} = \frac{\frac{\lambda}{\alpha Var[\theta|S_L]}\nu_L + \frac{(1-\lambda)}{\alpha Var[\theta|\nu_L]}E[\theta|\nu_L]}{\frac{\lambda}{\alpha Var[\theta|S_L]} + \frac{(1-\lambda)}{\alpha Var[\theta|\nu_L]}}. \tag{111}$$

Plugging equation 111, the model parameters and $Z_{U,H}$ in 110 yields

$$E\left[E[V(W_{1U})|\nu_H]\right] = -exp\left[-\alpha RW_{0U}\right] *$$
$$E\left[exp\left[-\frac{\alpha^4\lambda^2\sigma_{\epsilon_L}^6\sigma_u^4\left(\lambda^2+\alpha^2\sigma_{\epsilon_L}^2\sigma_u^2\right)(\sigma_{\epsilon_L}^2+\sigma_\theta^2)^2}{2(\lambda^2\sigma_{\epsilon_L}^2+\alpha^2\sigma_{\epsilon_L}^4\sigma_u^2+\lambda^2\sigma_\theta^2)\left(\sigma_{\epsilon_L}^2\left(\lambda^2+\alpha^2\sigma_{\epsilon_L}^2\sigma_u^2\right)\sigma_\theta+\lambda\left(\lambda+\alpha^2\sigma_{\epsilon_l}^2\sigma_u^2\right)\sigma_\theta^3\right)^2} * Var[\nu_H] * Z_{U,H}^2\right]\right], \tag{112}$$

and

$$t_L = -\frac{\alpha^4\sigma_{\epsilon_L}^6\sigma_u^4\left(\lambda^2+\alpha^2\sigma_{\epsilon_L}^2\sigma_u^2\right)(\sigma_{\epsilon_L}^2+\sigma_\theta^2)^2}{2\left(\sigma_{\epsilon_L}^2\left(\lambda^2+\alpha^2\sigma_{\epsilon_L}^2\sigma_u^2\right)\sigma_\theta+\lambda\left(\lambda+\alpha^2\sigma_{\epsilon_l}^2\sigma_u^2\right)\sigma_\theta^3\right)^2}. \tag{113}$$

The corresponding $t = t_H$ for the high-variance regime is given analogous to 113 and writes

$$t_H = -\frac{\alpha^4\sigma_{\epsilon_H}^6\sigma_u^4\left(\lambda^2+\alpha^2\sigma_{\epsilon_H}^2\sigma_u^2\right)(\sigma_{\epsilon_H}^2+\sigma_\theta^2)^2}{2\left(\sigma_{\epsilon_H}^2\left(\lambda^2+\alpha^2\sigma_{\epsilon_H}^2\sigma_u^2\right)\sigma_\theta+\lambda\left(\lambda+\alpha^2\sigma_{\epsilon_l}^2\sigma_u^2\right)\sigma_\theta^3\right)^2}. \tag{114}$$

Using 94, 109 and 113 the overall utility of the uninformed playing the more aggressive strategy (L) is given by

$$E[V(W_{1U,L})] = V(RW_{0U})\left(\frac{p}{\sqrt{1+2t_{H,L}}} + \frac{1-p}{\sqrt{1+2t_L}}\right), \tag{115}$$

while, according to 99 and 114 the overall utility for playing the more modest strategy (H) is given by

$$E[V(W_{1U,H})] = V(RW_{0U})\left(\frac{p}{\sqrt{1+2t_H}} + \frac{1-p}{\sqrt{1+2t_{L,H}}}\right). \tag{116}$$

As 94 is a convex function $\forall\, t < 0$ and the sum of a convex function is also a convex function, 115 and 116 intersect at most once and they do. Thus, there exists a unique value of p, which we denote as p^*, dependent on the parameters of the model, which decides the optimal strategy of the uninformed. The value of $p*$ is determined by the equation

$$E[V(W_{1U,L})] = E[V(W_{1U,H})]$$
$$\left(\frac{p}{\sqrt{1+2t_{H,L}}} + \frac{1-p}{\sqrt{1+2t_L}}\right) = \left(\frac{p}{\sqrt{1+2t_H}} + \frac{1-p}{\sqrt{1+2t_{L,H}}}\right). \tag{117}$$

Solving for p yields

$$p^* = \frac{\frac{1}{\sqrt{1+2t_{H,L}}} - \frac{1}{\sqrt{1+2t_L}}}{-\frac{1}{\sqrt{1+2t_H}} + \frac{1}{\sqrt{1+2t_{H,L}}} - \frac{1}{\sqrt{1+2t_L}} + \frac{1}{\sqrt{1+2t_{L,H}}}}. \tag{118}$$

As long as $p < p^*$, it is optimal for the uninformed agent to play the more aggressive strategy (L). As soon as $p > p^*$, it is optimal for the agent to play the more conservative strategy (H). For $p = p^*$, the uninformed agent is indifferent between the two strategies.

□

A.5 Proof of Theorem 4.1

Proof. Given that the uninformed plays strategy (L), the price movement occurs in the high-variance regime. The first-period price P_1 is given by 103, while the price in the second period P_2 is given by 35. Plugging in the parameters and rearranging the price difference between period one and two, $P_2 - P_1 = \Delta P_2$ can be written as

$$\Delta P_2 = \\ -\frac{(1-\lambda)\lambda^2\alpha^2\sigma^2_{\epsilon_H}\sigma^2_u(\sigma^2_{\epsilon_H} - \sigma^2_{\epsilon_L})\left(\sigma^2_{\epsilon_H} + \sigma^2_\theta\right)}{\left(\sigma^2_{\epsilon_H}\left(\lambda^2 + \alpha^2\sigma^2_{\epsilon_L}\sigma^2_u\right) + \lambda\left(\lambda + \alpha^2\sigma^2_{\epsilon_l}\sigma^2_u\right)\sigma^2_\theta\right)\left(\alpha^2\sigma^4_{\epsilon_H}\sigma^2_u + \lambda^2\sigma^2_\theta + \lambda\sigma^2_{\epsilon_H}\left(\lambda + \alpha^2\sigma^2_u\sigma^2_\theta\right)\right)} \cdot (\nu_H - \hat{\theta}). \tag{119}$$

Hence, it holds $\Delta P_2 < 0 \quad \forall \quad \nu_H > \hat{\theta}$ and $\Delta P_2 > 0 \quad \forall \quad \nu_H < \hat{\theta}$. This translates into a negative covariance of the price change between t_1 and t_2. Given $E[\Delta P_2] = 0$ the covariance is determined by $E[\Delta P_1 \Delta P_2]$ which writes

$$cov[\Delta P_1 \Delta P_2] = \\ -\frac{(1-\lambda)\lambda^3\alpha^2\sigma^2_{\epsilon_H}\sigma^2_u(\sigma^2_{\epsilon_H} - \sigma^2_{\epsilon_L})\left(\sigma^2_{\epsilon_H} + \sigma^2_\theta\right)^2}{\left(\sigma^2_{\epsilon_H}\left(\lambda^2 + \alpha^2\sigma^2_{\epsilon_L}\sigma^2_u\right) + \lambda\left(\lambda + \alpha^2\sigma^2_{\epsilon_l}\sigma^2_u\right)\sigma^2_\theta\right)^2\left(\alpha^2\sigma^4_{\epsilon_H}\sigma^2_u + \lambda^2\sigma^2_\theta + \lambda\sigma^2_{\epsilon_H}\left(\lambda + \alpha^2\sigma^2_u\sigma^2_\theta\right)\right)} \cdot Var[\nu_H] \tag{120}$$

One can infer from equation 120 that the covariance in the price movement if the uninformed plays L is always negative, $cov[\Delta P_1 \Delta P_2] < 0$.

Given that the uninformed plays strategy (H) modest, the price movement occurs in the low-variance regime and the mechanics are vice versa. The first-period price P_1 is given by 38, while the price in the second period P_2 is given by 111. Plugging in the parameters and rearranging the price difference between period one and two, $P_2 - P_1 = \Delta P_2$ can be written as

$$\Delta P_2 = \frac{(1-\lambda)\lambda^2\alpha^2\sigma_{\epsilon_L}^2\sigma_u^2(\sigma_{\epsilon_H}^2 - \sigma_{\epsilon_L}^2)\left(\sigma_{\epsilon_L}^2 + \sigma_\theta^2\right)}{\left(\sigma_{\epsilon_L}^2\left(\lambda^2 + \alpha^2\sigma_{\epsilon_H}^2\sigma_u^2\right) + \lambda\left(\lambda + \alpha^2\sigma_{\epsilon_H}^2\sigma_u^2\right)\sigma_\theta^2\right)\left(\alpha^2\sigma_{\epsilon_L}^4\sigma_u^2 + \lambda^2\sigma_\theta^2 + \lambda\sigma_{\epsilon_L}^2\left(\lambda + \alpha^2\sigma_u^2\sigma_\theta^2\right)\right)} \cdot (\nu_L - \hat{\theta}). \tag{121}$$

Hence it holds $\Delta P_2 > 0 \quad \forall \quad \nu_L > \hat{\theta}$ and $\Delta P_2 < 0 \quad \forall \quad \nu_L < \hat{\theta}$. This translates into a positive covariance of the price change between t_1 and t_2. Given $E[\Delta P_2] = 0$, the covariance is determined by $E[\Delta P_1 \Delta P_2]$ which writes

$$cov[\Delta P_1 \Delta P_2] = \frac{(1-\lambda)\lambda^3\alpha^2\sigma_{\epsilon_L}^2\sigma_u^2(\sigma_{\epsilon_H}^2 - \sigma_{\epsilon_L}^2)\left(\sigma_{\epsilon_L}^2 + \sigma_\theta^2\right)^2}{\left(\sigma_{\epsilon_L}^2\left(\lambda^2 + \alpha^2\sigma_{\epsilon_H}^2\sigma_u^2\right) + \lambda\left(\lambda + \alpha^2\sigma_{\epsilon_H}^2\sigma_u^2\right)\sigma_\theta^2\right)^2\left(\alpha^2\sigma_{\epsilon_L}^4\sigma_u^2 + \lambda^2\sigma_\theta^2 + \lambda\sigma_{\epsilon_L}^2\left(\lambda + \alpha^2\sigma_u^2\sigma_\theta^2\right)\right)} \cdot Var[\nu_H] \tag{122}$$

One can infer from equation 122 that the covariance in the price movement if the uninformed plays H is always positive, $cov[\Delta P_1 \Delta P_2] > 0$.

□

A.6 Proof of Proposition 1

Proof. Notice that 57 is a weighted sum of two moment-generating functions (as shown in 3.4, the expected utility of a CARA investor is essentially a moment-generating function). Every moment-generating function (MGF) is continuously differentiable in its domain of existence. It is known that the sum of continuously-differentiable functions is also continuously differentiable and the derivative of a continuously-differentiable function is continuous. Hence, 59 is continuous and the intermediate value theorem is applicable. Monotonicity is implied by the fact that the first derivative of 59 w.r.t X_U is given by

$$
\begin{aligned}
&-\alpha^2 exp\Bigg(\frac{1}{2}\alpha\big(-4RW_{0U}+X_U(-2((E[\theta|S_L]-RP)+(E[\theta|S_H]-RP))\\
&+\alpha X_U\left(Var[\theta|S_L]+Var[\theta|S_H]\right)))\Bigg)\\
&\Bigg(\omega_H\left(Var[\theta|S_H]+((E[\theta|S_H]-RP)-\alpha X_U Var[\theta|S_H])^2\right)\\
&exp\left(-\alpha\left(RW_{0U}+(E[\theta|S_L]-RP)X_U-\frac{\alpha X_U^2}{2}Var[\theta|S_L]\right)\right)\\
&+\omega_L\left(Var[\theta|S_L]+((E[\theta|S_L]-RP)-\alpha X_U Var[\theta|S_L])^2\right)\\
&exp\left(-\alpha\left(RW_{0U}+(E[\theta|S_H]-RP)X_U-\frac{\alpha X_U^2}{2}Var[\theta|S_H]\right)\right)\Bigg).
\end{aligned}
\tag{123}
$$

The above expression is always smaller than 0 and thus, 59 is monotonically decreasing in X_U. Our next goal is to prove that the optimal demand of the uninformed agents is bounded by the optimal demand of the informed agents $X_U \in (X_{I,H}, X_{I,L})$. This is achieved in two steps: first, by substituting the demand of the uninformed agents by the optimal demand of the informed agents; and second, by verifying that the ratio of $\frac{FOC(X_{I,H})}{FOC(X_{I,L})} < 0$. We define the agent's utility in the respective regimes as a function of S_i and X_U with

$$
\Omega(S_i, X_U) = -\alpha\left(RW_{0U}+(E[\theta|S_i]-RP)X_U-\frac{\alpha X_U^2}{2}Var[\theta|S_i]\right). \tag{124}
$$

Substituting $X_U = X_{I,H}$ into the FOC, one gets

$$
\omega_L\cdot\alpha\left((E[\theta|S_L]-RP)-(E[\theta|S_H]-RP)\frac{Var[\theta|S_L]}{Var[\theta|S_H]}\right)\cdot exp\left(\Omega\left(S_i, X_U = X_{I,H}\right)\right) \tag{125}
$$

which, after plugging in all parameters, reduces to

$$
\omega_L\cdot\alpha\left(\frac{\sigma_\theta^2(\sigma_{\epsilon_H}^2-\sigma_{\epsilon_L}^2)}{\sigma_H^2(\sigma_\theta^2+\sigma_{\epsilon_L}^2)}(S-RP)\right)\cdot exp\left(\Omega\left(S_i, X_U = X_{I,H}\right)\right). \tag{126}
$$

Since the exponential part of the expression is always positive, it is sufficient to analyze the expression in front of the exponential function. Taking a closer look and knowing that $\sigma_{\epsilon_H}^2 > \sigma_{\epsilon_L}^2$ and due to risk aversion $|S| > |P|$, it becomes apparent that the above expression is > 0 for $S > 0$ and < 0 for $S < 0$.
Substituting $X_U = X_{IL}$ the dynamics are exactly vice versa, yielding

$$
\omega_H\cdot\alpha\left((E[\theta|S_H]-RP)-(E[\theta|S_L]-RP)\frac{Var[\theta|S_H]}{Var[\theta|S_L]}\right)\cdot exp\left(\Omega\left(S_i, X_U = X_{I,L}\right)\right), \tag{127}
$$

which simplifies to

$$\omega_H \cdot \alpha \left(\frac{\sigma_\theta^2(\sigma_{\epsilon_L}^2 - \sigma_{\epsilon_H}^2)}{\sigma_L^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2)} (S - RP) \right) \cdot exp\left(\Omega\left(S_i, X_U = X_{I,L}\right)\right), \tag{128}$$

which is > 0 for $S < 0$ and < 0 for $S > 0$. Combining equations 128 and 126 in a ratio and simplifying

$$\begin{aligned} &\frac{\omega_L \cdot \alpha \left(\frac{\sigma_\theta^2(\sigma_{\epsilon_H}^2 - \sigma_{\epsilon_L}^2)}{\sigma_H^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2)} (S - RP) \right) \cdot exp\left(\Omega\left(S_i, X_U = X_{I,H}\right)\right)}{\omega_H \cdot \alpha \left(\frac{\sigma_\theta^2(\sigma_{\epsilon_L}^2 - \sigma_{\epsilon_H}^2)}{\sigma_L^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2)} (S - RP) \right) \cdot exp\left(\Omega\left(S_i, X_U = X_{I,L}\right)\right)} \\ &= \frac{\omega_L \cdot \sigma_L^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2)(\sigma_{\epsilon_H}^2 - \sigma_{\epsilon_L}^2) \cdot exp\left(\Omega\left(S_i, X_U = X_{I,H}\right)\right)}{\omega_H \cdot \sigma_H^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2)(\sigma_{\epsilon_L}^2 - \sigma_{\epsilon_H}^2) \cdot exp\left(\Omega\left(S_i, X_U = X_{I,L}\right)\right)} \\ &= -\frac{\omega_L \cdot \sigma_L^2(\sigma_\theta^2 + \sigma_{\epsilon_H}^2) \cdot exp\left(\Omega\left(S_i, X_U = X_{I,H}\right)\right)}{\omega_H \cdot \sigma_H^2(\sigma_\theta^2 + \sigma_{\epsilon_L}^2) \cdot exp\left(\Omega\left(S_i, X_U = X_{I,L}\right)\right)}, \end{aligned} \tag{129}$$

it becomes evident that the above ratio is always < 0. This makes apparent that 0 is always in between the two function values $FOC(X_{I,l})$ and $FOC(X_{I,H})$.

The intermediate value theorem states that

> *"If f is continuous on a closed interval $[a, b]$, and c is any number between $f(a)$ and $f(b)$ inclusive, then there is at least one number x in the closed interval such that $f(x) = c$ if a continuous function, f, with an interval, $[x, y]$, as its domain takes values $f(x)$ and $f(y)$ at each end of the interval, then it also takes any value between $f(x)$ and $f(y)$ at some point within the interval."*

Putting together all three parts of the proof, there exists a value of X_U, in a space bounded by $X_{I,l}$ and $X_{I,H}$ for which $FOC(X_U) = 0$. As $FOC(X_U)$ is monotone, 0 is crossed only once and the optimal value of X_U maximizing the agents utility is unique. □

A.7 Proof of Proposition 2

Proof. Defining the uninformed agents' utility in the respective regimes as a function of S_i and $P_{\lambda,i}$ with

$$\begin{aligned} \Lambda_L(S_i, X_U(P_{\lambda,i})) &= -\alpha \left(RW_{0U} + (E[\theta|S_L] - RP)X_U(P_{\lambda,i}) - \frac{\alpha X_U(P_{\lambda,i})^2}{2} Var[\theta|S_L] \right) \\ \Lambda_H(S_i, X_U(P_{\lambda,i})) &= -\alpha \left(RW_{0U} + (E[\theta|S_H] - RP)X_U(P_{\lambda,i}) - \frac{\alpha X_U(P_{\lambda,i})^2}{2} Var[\theta|S_H] \right) \end{aligned} \tag{130}$$

given $S = S_H$ plugging the indirect demand into the FOC, one gets

$$0 = \omega_L \cdot \left(\alpha\left((E[\theta|S_L] - RP) - \alpha Var[\theta|S_L]\left[\frac{u}{1-\lambda} - \frac{\lambda}{1-\lambda}\frac{E[\theta|S_H] - RP}{\alpha Var[\theta|S_H]}\right]\right)\right) exp\left(\Lambda_L(S_H, P_{\lambda,H})\right)$$
$$+\omega_H \cdot \left(\alpha\left((E[\theta|S_H] - RP) - \alpha Var[\theta|S_H]\left[\frac{u}{1-\lambda} - \frac{\lambda}{1-\lambda}\frac{E[\theta|S_H] - RP}{\alpha Var[\theta|S_H]}\right]\right)\right) exp\left(\Lambda_H(S_H, P_{\lambda,H})\right) \tag{131}$$

which can be reduced to

$$0 = \frac{\omega_L \cdot \alpha}{1-\lambda}\left((1-\lambda)(E[\theta|S_L] - RP) + \frac{\lambda Var[\theta|S_L]}{Var[\theta|S_H]}(E[\theta|S_H] - RP) - \alpha u Var[\theta|S_L]\right) exp\left(\Lambda_L(S_H, P_{\lambda,H})\right)$$
$$+ \frac{\omega_H \cdot \alpha}{1-\lambda}\left((E[\theta|S_H] - RP) - \alpha u Var[\theta|S_H]\right) exp\left(\Lambda_H(S_H, P_{\lambda,H})\right). \tag{132}$$

Simplifying and collecting the $P_{\lambda,i}$ that are not in the exponential on one side yields an implicit equilibrium price function for the scenario in which $S_i = S_H$

$$P_{\lambda,H} = \omega_L \frac{\left((1-\lambda)E[\theta|S_L] + \frac{\lambda Var[\theta|S_L]}{Var[\theta|S_H]}(E[\theta|S_H] - \alpha u Var[\theta|S_L]\right) \cdot exp\left(\Lambda_L(S_H, P_{\lambda,H})\right)}{\left(\omega_L \frac{Var[\theta|S_H] - \lambda(Var[\theta|S_H] - Var[\theta|S_L])}{Var[\theta|S_H]} \cdot exp\left(\Lambda_L(S_H, P_{\lambda,H})\right) + \omega_H \cdot exp\left(\Lambda_H(S_H, P_{\lambda,H})\right)\right)}$$
$$+ \omega_H \frac{(E[\theta|S_H] - \alpha u Var[\theta|S_H]) \cdot exp\left(\Lambda_H(S_H, P_{\lambda,H})\right)}{\left(\omega_L \frac{Var[\theta|S_H] - \lambda(Var[\theta|S_H] - Var[\theta|S_L])}{Var[\theta|S_H]} \cdot exp\left(\Lambda_L(S_H, P_{\lambda,H})\right) + \omega_H \cdot exp\left(\Lambda_H(S_H, P_{\lambda,H}).\right)\right)} \tag{133}$$

Plugging in $P_{\lambda,H} = \hat{P_{\lambda,H}} = \frac{1}{R}((E[\theta|S_H] - \alpha u Var[\theta|S_H])$ yields

$$0 = \frac{\omega_L \cdot \alpha}{1-\lambda}\left((1-\lambda)(E[\theta|S_L] - E[\theta|S_H]) + (1-\lambda)\alpha u(Var[\theta|S_H] - Var[\theta|S_L])\right) exp\left(\Lambda_L(S_H, \hat{P_{\lambda,H}})\right)$$
$$0 = \omega_L \cdot \alpha\left((E[\theta|S_L] - E[\theta|S_H]) + \alpha u(Var[\theta|S_H] - Var[\theta|S_L])\right) exp\left(\Lambda_L(S_H, \hat{P_{\lambda,H}})\right). \tag{134}$$

Plugging in $P_{\lambda,H} = \hat{P_{\lambda,L}} = \frac{1}{R}((E[\theta|S_L] - \alpha u Var[\theta|S_L])$ results in

$$0 = \frac{\omega_L \cdot \alpha}{1-\lambda}\lambda\frac{Var[\theta|S_L]}{Var[\theta|S_H]}\left((E[\theta|S_H] - E[\theta|S_L]) - \alpha u(Var[\theta|S_H] - Var[\theta|S_L])\right) exp\left(\Lambda_L(S_H, \hat{P_{\lambda,L}})\right)$$
$$+ \frac{\omega_H \cdot \alpha}{1-\lambda}\left((E[\theta|S_H] - E[\theta|S_L]) - \alpha u(Var[\theta|S_H] - Var[\theta|S_L])\right) exp\left(\Lambda_H(S_H, \hat{P_{\lambda,L}})\right). \tag{135}$$

Setting up the ratio between 135 and 134 it becomes apparent that the two prices under perfect information constitute the bounds of the equilibrium price

$$\frac{\frac{\omega_L \cdot \alpha}{1-\lambda}\lambda\frac{Var[\theta|S_L]}{Var[\theta|S_H]}\left((E[\theta|S_H]-E[\theta|S_L])-\alpha u(Var[\theta|S_H]-Var[\theta|S_L])\right) exp\left(\Lambda_L(S_H,\hat{P_{\lambda,L}})\right)}{\omega_L \cdot \alpha\left((E[\theta|S_L]-E[\theta|S_H])+\alpha u(Var[\theta|S_H]-Var[\theta|S_L])\right) exp\left(\Lambda_L(S_H,\hat{P_{\lambda,H}})\right)}$$
$$+\frac{\frac{\omega_H \cdot \alpha}{1-\lambda}\left((E[\theta|S_H]-E[\theta|S_L])-\alpha u(Var[\theta|S_H]-Var[\theta|S_L])\right) exp\left(\Lambda_H(S_H,\hat{P_{\lambda,L}})\right)}{\omega_L \cdot \alpha\left((E[\theta|S_L]-E[\theta|S_H])+\alpha u(Var[\theta|S_H]-Var[\theta|S_L])\right) exp\left(\Lambda_L(S_H,\hat{P_{\lambda,H}})\right)}, \tag{136}$$

which can be rearranged as

$$\frac{-\frac{\lambda}{1-\lambda}\frac{Var[\theta|S_L]}{Var[\theta|S_H]}\left((E[\theta|S_L]-E[\theta|S_H])+\alpha u(Var[\theta|S_H]-Var[\theta|S_L])\right) exp\left(\Lambda_L(S_H,\hat{P_{\lambda,L}})\right)}{\left((E[\theta|S_L]-E[\theta|S_H])+\alpha u(Var[\theta|S_H]-Var[\theta|S_L])\right) exp\left(\Lambda_L(S_H,\hat{P_{\lambda,H}})\right)}$$
$$+\frac{-\frac{\omega_H}{1-\lambda}\left((E[\theta|S_L]-E[\theta|S_H])+\alpha u(Var[\theta|S_H]-Var[\theta|S_L])\right) exp\left(\Lambda_H(S_H,\hat{P_{\lambda,L}})\right)}{\omega_L\left((E[\theta|S_L]-E[\theta|S_H])+\alpha u(Var[\theta|S_H]-Var[\theta|S_L])\right) exp\left(\Lambda_L(S_H,\hat{P_{\lambda,H}})\right)}, \tag{137}$$

and simplified to

$$-\frac{1}{1-\lambda}\left(\lambda\frac{Var[\theta|S_L]}{Var[\theta|S_H]}\cdot\frac{exp\left(\Lambda_L(S_H,\hat{P_{\lambda,L}})\right)}{exp\left(\Lambda_L(S_H,\hat{P_{\lambda,H}})\right)}+\frac{\omega_H}{\omega_L}\cdot\frac{exp\left(\Lambda_H(S_H,\hat{P_{\lambda,L}})\right)}{exp\left(\Lambda_L(S_H,\hat{P_{\lambda,H}})\right)}\right). \tag{138}$$

Adapting the identical approach for $S = S_L$

$$X_U(P_{\lambda,L}) = \frac{u}{1-\lambda} - \frac{\lambda}{1-\lambda}\frac{E[\theta|S_L]-RP_{\lambda,L}}{\alpha Var[\theta|S_L]}. \tag{139}$$

Analogous to above plugging the indirect demand into the FOC, one gets

$$0=\omega_L\cdot\left(\alpha\left((E[\theta|S_L]-RP)-\alpha Var[\theta|S_L]\left[\frac{u}{1-\lambda}-\frac{\lambda}{1-\lambda}\frac{E[\theta|S_L]-RP}{\alpha Var[\theta|S_L]}\right]\right)\right) exp\left(\Lambda_L(S_L,P_{\lambda,L})\right)$$
$$+\omega_H\cdot\left(\alpha\left((E[\theta|S_H]-RP)-\alpha Var[\theta|S_H]\left[\frac{u}{1-\lambda}-\frac{\lambda}{1-\lambda}\frac{E[\theta|S_L]-RP}{\alpha Var[\theta|S_L]}\right]\right)\right) exp\left(\Lambda_H(S_L,P_{\lambda,L})\right), \tag{140}$$

which simplifies to

$$0=\frac{\omega_L\cdot\alpha}{1-\lambda}\left((E[\theta|S_L]-RP)-\alpha u Var[\theta|S_L]\right) exp\left(\Lambda_L(S_L,P_{\lambda,L})\right)$$
$$+\frac{\omega_H\cdot\alpha}{1-\lambda}\left((1-\lambda)(E[\theta|S_H]-RP)+\lambda\frac{Var[\theta|S_H]}{Var[\theta|S_L]}(E[\theta|S_L]-RP)-\alpha u Var[\theta|S_H]\right) exp\left(\Lambda_H(S_L,P_{\lambda,L})\right).$$

$$(141)$$

Plugging in $P_{\lambda,L}=\hat{P_{\lambda,L}}=\frac{1}{R}((E[\theta|S_L]-\alpha uVar[\theta|S_L])$ one gets

$$\begin{aligned} 0 &= \frac{\omega_H\cdot\alpha}{1-\lambda}\left((1-\lambda)(E[\theta|S_H]-E[\theta|S_L])+(1-\lambda)\alpha u(Var[\theta|S_L]-Var[\theta|S_H])\right) exp\left(\Lambda_H(S_L,\hat{P_{\lambda,L}})\right) \\ &= \omega_H\cdot\alpha\left((E[\theta|S_H]-E[\theta|S_L])+\alpha u(Var[\theta|S_L]-Var[\theta|S_H])\right) exp\left(\Lambda_H(S_L,\hat{P_{\lambda,L}})\right). \end{aligned} \tag{142}$$

Now using $P_{\lambda,L}=\hat{P_{\lambda,H}}=\frac{1}{R}((E[\theta|S_H]-\alpha uVar[\theta|S_H])$ results in

$$\begin{aligned} 0 &= \frac{\omega_L\cdot\alpha}{1-\lambda}\left((E[\theta|S_L]-E[\theta|S_H])+\alpha u(Var[\theta|S_H]-Var[\theta|S_L])\right) exp\left(\Lambda_L(S_L,\hat{P_{\lambda,H}})\right) \\ &+\frac{\omega_H\cdot\alpha}{1-\lambda}\frac{\lambda Var[\theta|S_H]}{Var[\theta|S_L]}\left((E[\theta|S_L]-E[\theta|S_H])+\alpha u(Var[\theta|S_H]-Var[\theta|S_L])\right) exp\left(\Lambda_H(S_L,\hat{P_{\lambda,H}})\right). \end{aligned} \tag{143}$$

Using rations again

$$\begin{aligned} &= \frac{\frac{\omega_L\cdot\alpha}{1-\lambda}\left((E[\theta|S_L]-E[\theta|S_H])+\alpha u(Var[\theta|S_H]-Var[\theta|S_L])\right) exp\left(\Lambda_L(S_L,\hat{P_{\lambda,H}})\right)}{\omega_H\cdot\alpha\left((E[\theta|S_H]-E[\theta|S_L])+\alpha u(Var[\theta|S_L]-Var[\theta|S_H])\right) exp\left(\Lambda_H(S_L,\hat{P_{\lambda,L}})\right)} \\ &+\frac{\frac{\omega_H\cdot\alpha}{1-\lambda}\frac{\lambda Var[\theta|S_H]}{Var[\theta|S_L]}\left((E[\theta|S_L]-E[\theta|S_H])+\alpha u(Var[\theta|S_H]-Var[\theta|S_L])\right) exp\left(\Lambda_H(S_L,\hat{P_{\lambda,H}})\right)}{\omega_H\cdot\alpha\left((E[\theta|S_H]-E[\theta|S_L])+\alpha u(Var[\theta|S_L]-Var[\theta|S_H])\right) exp\left(\Lambda_H(S_L,\hat{P_{\lambda,L}})\right)} \end{aligned} \tag{144}$$

$$\begin{aligned} &\frac{-\frac{\lambda}{1-\lambda}\frac{Var[\theta|S_H]}{Var[\theta|S_L]}\left((E[\theta|S_H]-E[\theta|S_L])+\alpha u(Var[\theta|S_L]-Var[\theta|S_H])\right) exp\left(\Lambda_H(S_L,\hat{P_{\lambda,H}})\right)}{\left((E[\theta|S_H]-E[\theta|S_L])+\alpha u(Var[\theta|S_L]-Var[\theta|S_H])\right) exp\left(\Lambda_H(S_L,\hat{P_{\lambda,L}})\right)} \\ &+\frac{-\frac{\omega_L}{1-\lambda}\left((E[\theta|S_H]-E[\theta|S_L])+\alpha u(Var[\theta|S_L]-Var[\theta|S_H])\right) exp\left(\Lambda_L(S_L,\hat{P_{\lambda,H}})\right)}{\omega_h\left((E[\theta|S_H]-E[\theta|S_L])+\alpha u(Var[\theta|S_L]-Var[\theta|S_H])\right) exp\left(\Lambda_H(S_L,\hat{P_{\lambda,L}})\right)} \end{aligned} \tag{145}$$

which simplifies to

$$-\frac{1}{1-\lambda}\left(\lambda\frac{Var[\theta|S_H]}{Var[\theta|S_L]}\cdot\frac{exp\left(\Lambda_H(S_L,\hat{P_{\lambda,H}})\right)}{exp\left(\Lambda_H(S_L,\hat{P_{\lambda,L}})\right)}+\frac{\omega_L}{\omega_H}\cdot\frac{exp\left(\Lambda_L(S_L,\hat{P_{\lambda,H}})\right)}{exp\left(\Lambda_H(S_L,\hat{P_{\lambda,L}})\right)}\right). \tag{146}$$

The function implicitly characterizing the equilibrium demand 63 is again the derivative of a weighted sum of two MGFs and hence continuous. As both ratios, 138 as well as 146 are always negative, by the intermediate value theorem, the equilibriumpricefunction has to cross zero at least once, and the equilibrium price has to be bounded by the full information prices in each scenario.

Looking at 134, 135, 143 and 142, notice that the implicit price function is always positive for $P_{\lambda,i} = P_{\hat{\lambda},H}$ for all $(S_i - \theta) > -\alpha u\sigma_\theta^2$, while it is always negative for $P_{\lambda,i} = P_{\hat{\lambda},L}$ for all $(S_i - \theta) < \alpha u\sigma_\theta^2$ in both signal regimes. Knowing that $P_{\hat{\lambda},L} > P_{\hat{\lambda},H} \forall (S_i - \theta) > -\alpha u\sigma_\theta^2$, it becomes immediately apparent that $P_{\hat{\lambda},L}$ constitutes the upper bound of the price level, while $P_{\hat{\lambda},H}$ defines the lower bound. Given $(S_i - \theta) < -\alpha u\sigma_\theta^2$, the signs switch and $P_{\hat{\lambda},L}$ constitutes the lower bound while $P_{\hat{\lambda},H}$ the upper one. $\alpha u\sigma_\theta^2$ is the amount that compensates for the difference of the uncertainty discounts between the two full information prices given by $\alpha u(Var[\theta|S_H] - Var[\theta|S_L])$.

□